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ABSTRACT

Astronomical fundamentals are presented in this unit of the Project Physics text for use by senior high students. The geocentric system of Ptolemy is discussed in connection with Greek concepts including Aristarchus' heliocentric hypothesis. Analyses are made of Copernicus' reexamination, leading to Tycho's observations and compromise system. A new universe is introduced by using Kepler's laws and Galileo's work. Terrestrial and celestial dynamics are synthesized by reviewing Newton's concepts about motions under central forces, inverse-square law, and earthly motions; and application of Newton's theories to heavenly events are discussed. The text unit is concluded by an explanation of the making of theories. Historical developments are stressed to help students understand the way in which science influences man's activities. Also included in the unit are a chart of renowned people's life spans in the emerging Renaissance culture and tables of planets and their satellites. Problems with their answers are provided in two categories: study guide and end of section questions. Besides illustrations for explanation purposes, a glossary of general terms is given in the appendix. The work of Harvard Project Physics has been financially supported by: the Carnegie Corporation of New York, the Ford Foundation, the National Science Foundation, the Alfred P. Sloan Foundation, the United States Office of Education, and Harvard University. (CC)

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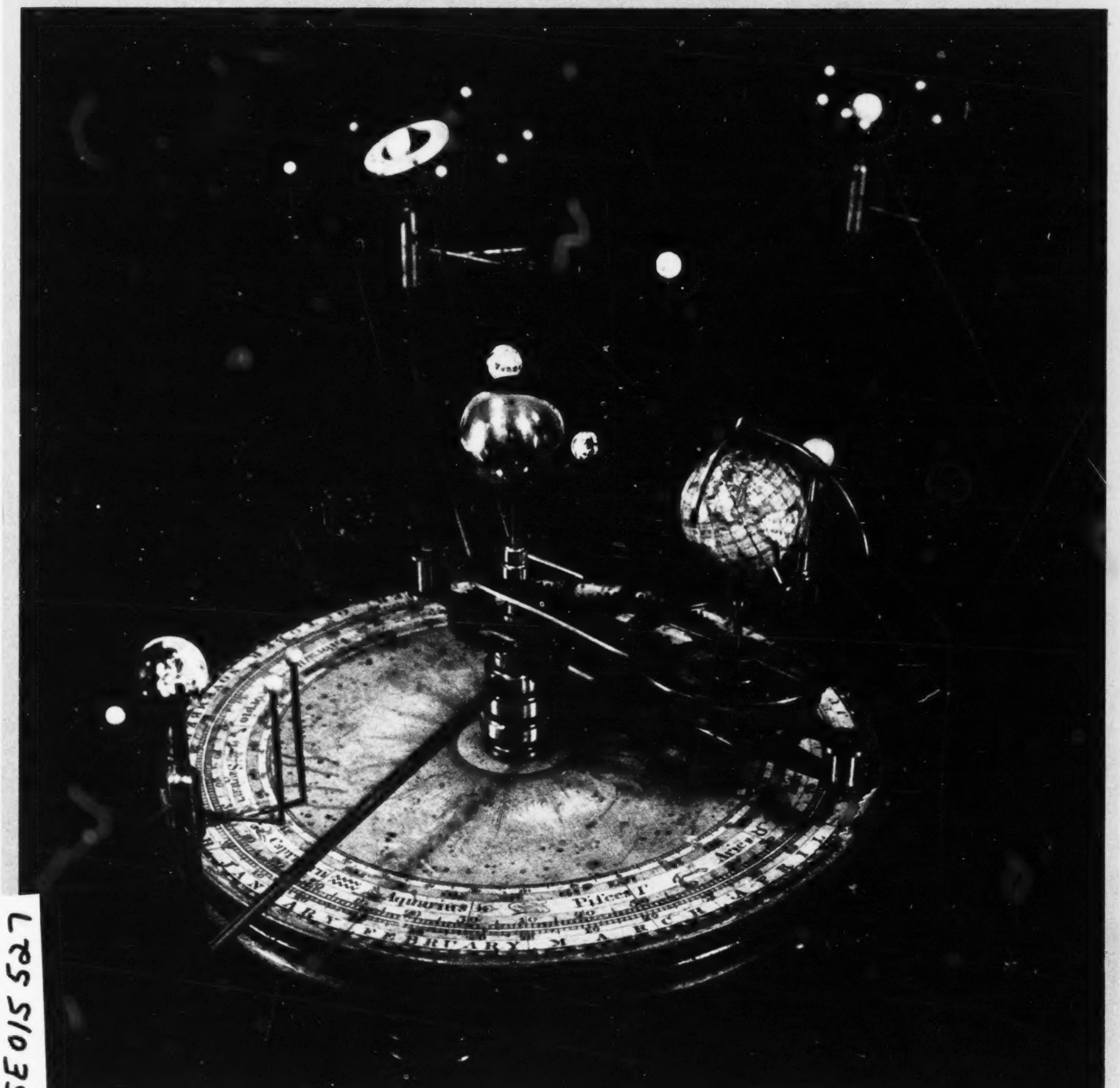
Project Physics **Text** 2

An Introduction to Physics

Motion in the Heavens



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Project Physics **Text**

An Introduction to Physics 2 Motion in the Heavens

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Welcome to the study of physics. This volume, more of a student's guide than a text of the usual kind, is part of a whole group of materials that includes a student handbook, laboratory equipment, films, programmed instruction, readers, transparencies, and so forth. Harvard Project Physics has designed the materials to work together. They have all been tested in classes that supplied results to the Project for use in revisions of earlier versions.

The Project Physics course is the work of about 200 scientists, scholars, and teachers from all parts of the country, responding to a call by the National Science Foundation in 1963 to prepare a new introductory physics course for nationwide use. Harvard Project Physics was established in 1964, on the basis of a two-year feasibility study supported by the Carnegie Corporation. On the previous pages are the names of our colleagues who helped during the last six years in what became an extensive national curriculum development program. Some of them worked on a full-time basis for several years; others were part-time or occasional consultants, contributing to some aspect of the whole course; but all were valued and dedicated collaborators who richly earned the gratitude of everyone who cares about science and the improvement of science teaching.

Harvard Project Physics has received financial support from the Carnegie Corporation of New York, the Ford Foundation, the National Science Foundation, the Alfred P. Sloan Foundation, the United States Office of Education and Harvard University. In addition, the Project has had the essential support of several hundred participating schools throughout the United States and Canada, who used and tested the course as it went through several successive annual revisions.

The last and largest cycle of testing of all materials is now completed; the final version of the Project Physics course will be published in 1970 by Holt, Rinehart and Winston, Inc., and will incorporate the final revisions and improvements as necessary. To this end we invite our students and instructors to write to us if in practice they too discern ways of improving the course materials.

The Directors
Harvard Project Physics

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The Aztec Calendar, carved over 100 years before our calendar was adopted, divides the year into eighteen months of twenty days each.

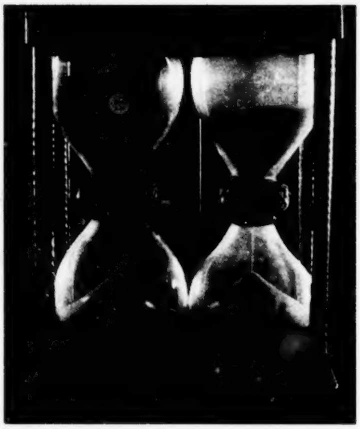
Prologue . Curiously, the oldest science, astronomy, deals with objects which are now known to be the most distant. Yet to you and me, as to the earliest observers of the skies, the sun, moon, planets and stars do not seem to be far away. On a clear night they seem so close that we can almost reach out and touch them.

Even in modern times outdoorsmen use the sun by day and the stars by night as a clock. Directions are indicated by the sun at rising and setting time, and true south can be determined from the position of the sun at noon. The pole star gives a bearing on true north after dark. The sun is also a crude calendar, its noontime altitude varying with the seasons.

The lives of the ancient people, and indeed of nearly all people who lived before electric lighting, were dominated by heavenly events. The working day began when the sun rose and ended when the sun set. Human activity was dominated by the presence or absence of daylight and the sun's warmth, changing season by season.

Over the centuries our clock has been devised so we can subdivide the days, and the calendar developed so we can record the passage of days into years. Of all the units used in regular life, "one day" is probably the most basic and surely the most ancient. For counting longer intervals, a "moon" or month was an obvious unit. But the moon is unsatisfactory as a timekeeper for establishing the agricultural year.

When, some 10,000 years ago, the nomadic tribes settled down to live in towns, they became dependent upon agriculture for their food. They needed a calendar for planning their plowing and sowing. Indeed, throughout recorded history most of the world's population has been involved in agriculture in the spring. If the crops were planted too early they might be killed by a frost, or the seeds rot in the ground. But if they were planted too late, the crops might not ripen before winter came. Therefore, a knowledge of the times for planting and harvesting had high survival value. A calendar for the agricultural year was very important. The making and improvement of the calendar were often the tasks of priests, who also set the dates for the religious festivals. Hence, the priests became the first astronomers.



Many of the great buildings of ancient times were constructed with careful astronomical orientation. The great pyramids of Egypt, tombs of the Pharaohs, have sides that run due north-south, and east-west. The impressive, almost frightening, circles of giant stones at Stonehenge in England appear to have been arranged about 2000 B.C. to permit accurate astronomical observations of the positions of the sun and moon. The Mayans and the Incas in America, as well as the Chinese, put enormous effort into buildings from which they could measure the changes in the positions of the

If you have never watched the night sky, start observations now. Chapters 5 and 6 are based on some simple observations of the sky.

Stonehenge, England, apparently a prehistoric observatory.

sun, moon and planets. Thus we know that for thousands of years men have carefully observed the motions of the heavenly bodies.

At least as early as 1000 B.C. the Babylonians and Egyptians had developed considerable ability in timekeeping. Their recorded observations are now being slowly unearthed. But the Egyptians, like the Mayans and others, were interested in practical forecasts and date-setting. To the best of our knowledge they did not try to explain their observations other than by tales and myths. Our western culture owes much to the efforts of the Egyptians and Babylonians who carefully recorded their observations of heavenly cycles. But our debt is greatest to the Greeks who began trying to explain what was seen.

The observations to be explained were begun out of idle curiosity, perhaps by shepherds to pass away the time, and later took on practical importance in calendar making. These simple observations became, during following centuries, the basis for one of the greatest of scientific triumphs: the work of Isaac Newton. The present unit tells how he united the study of motions on the earth with the study of motions in the heavens to produce a single universal science of motion.

Section of Babylonian clay tablet, now in the British Museum, records the positions of Jupiter from 132 B.C. to 60 B.C.

The Greeks recognized the contrast between forced and temporary motions on the earth and the unending cycles of motions in the heavens. About 600 B.C. they began to ask a new question: how can we explain these cyclic events in the sky in a simple way? What order and sense can we make of the heavenly happenings? The Greeks' answers, which are discussed in Chapter 5, had an important effect on science. For example, as we shall see, the writings of Aristotle, about 330 B.C., became widely studied and accepted in western Europe after 1200 A.D., and were important in the scientific revolution that followed.

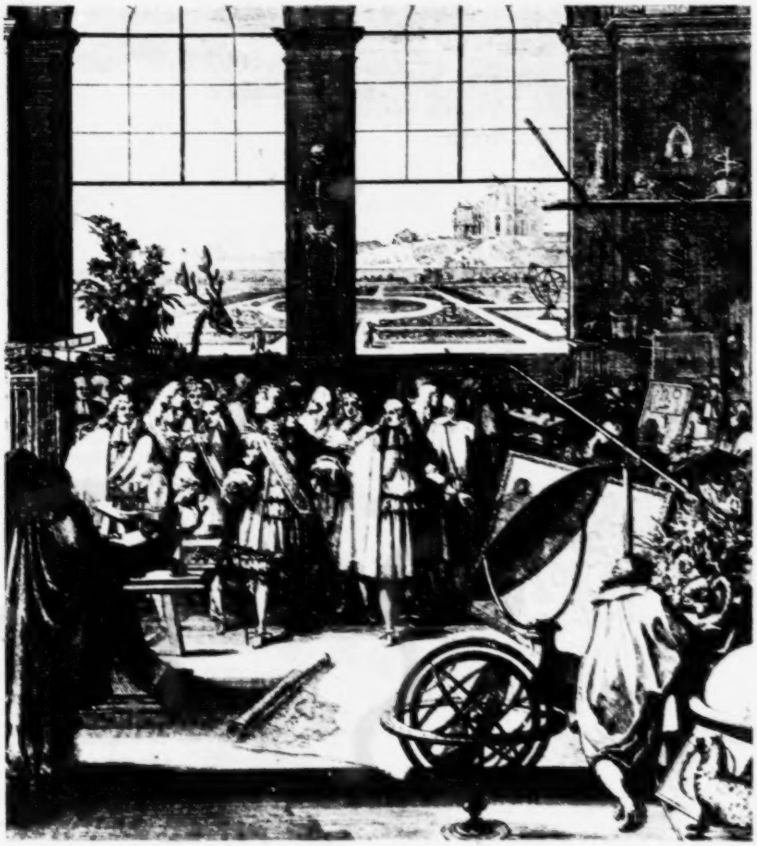
After the conquests of Alexander the Great, the center of Greek thought and science shifted to Egypt at the new city of Alexandria, founded in 332 B.C. There a great museum, actually similar to a modern research institute, was created about 290 B.C. and flourished for many centuries. But as the Greek civilization gradually declined, the practical-minded Romans captured Egypt, and interest in science died out. In 640 A.D. Alexandria was taken by the Moslems as they swept along the southern shore of the Mediterranean Sea and moved northward through Spain to the Pyrenees. Along the way they seized and preserved many libraries of Greek documents, some of which were later translated into Arabic and seriously studied. During the following centuries the Moslem scientists made new and better observations of the heavens, although they did not make major changes in the explanations or theories of the Greeks.

During this time, following the invasions by warring tribes from northern and central Europe, civilization in western Europe fell to a low level, and the works of the Greeks were forgotten. Eventually they were rediscovered by Europeans through Arabic translations found in Spain when the Moslems were forced out. By 1130 A.D. complete manuscripts of one of Aristotle's books were known in Italy and in Paris. After the founding of the University of Paris in 1170, many other writings of Aristotle were acquired and studied both there and at the new English Universities, Oxford and Cambridge.

During the next century, Thomas Aquinas (1225-1274) blended major elements of Greek thought and Christian theology into a single philosophy. This blend, known as Thomism, was widely accepted in western Europe for several centuries. In achieving this commanding and largely successful synthesis, Aquinas accepted the physics and astronomy of Aristotle. Because the science was blended with theology, any questioning of the science seemed also to be a questioning of the theology. Thus for a time there was little criticism of the Aristotelian science.

The Renaissance movement, which spread out across Europe from Italy, brought new art and new music. Also, it brought new ideas about the universe and man's place in it. Curiosity and a questioning attitude became acceptable, even prized. Men acquired a new confidence in their ability to learn about the world. Among those whose work introduced the new age were Columbus and Vasco da Gama, Gutenberg and da Vinci, Michelangelo and Dürer, Erasmus, Vesalius, and Agricola, Luther, and Henry VIII. The chart opposite page 29 shows their life spans. Within this emerging Renaissance culture lived Copernicus, whose reexamination of astronomical theories is discussed in Chapter 6.

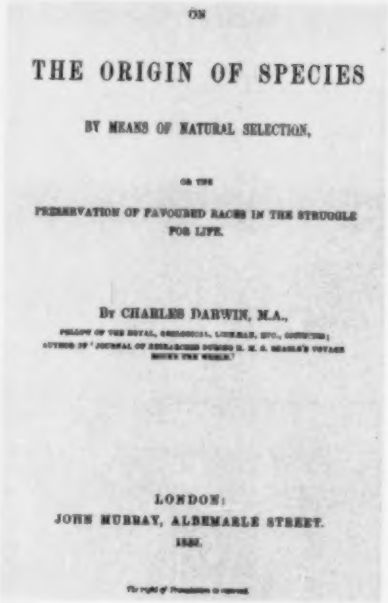
Further changes in astronomical theory were made by Kepler through his mathematics and by Galileo through his observations and writings, which are discussed in Chapter 7. In Chapter 8 we shall see that Newton's work in the second half of the seventeenth century extended the ideas about earthly motions so that they could also explain the motions observed in the heavens—a magnificent synthesis of terrestrial and celestial dynamics.



Louis XIV visiting the French Academy of Sciences, which he founded in the middle of the seventeenth century. Seen through the right-hand window is the Paris Observatory, then under construction.

Great scientific advances can affect ideas outside science. For example, Newton's impressive work helped to bring a new sense of confidence. Man now seemed capable of understanding all things in heaven and earth. This great change in attitude was a characteristic of the Age of Reason in the eighteenth century. To a degree, what we think today and how we run our affairs are still based on these events of three centuries ago.

The decisive changes in thought that developed at the start of the Renaissance within a period of a century can be compared to changes during the past hundred years. This period might extend from the publication in 1859 of Darwin's Origin of Species to the first large-scale release of atomic energy in 1945. Within this recent interval lived such scientists as Mendel and Pasteur, Planck and Einstein, Rutherford and Bohr. The ideas they and others introduced into science during the last century have had increasing importance. These scientific ideas are just as much a part of our time as the ideas and works of such persons as Roosevelt, Ghandi and Pope John XXIII; Marx and Lenin; Freud and Dewey; Picasso and Stravinsky; or Shaw and Joyce. If, therefore, we understand the way in which science influenced the men of past centuries, we shall be better prepared to understand how science influences our thought and lives today.



ON

THE ORIGIN OF SPECIES

BY MEANS OF NATURAL SELECTION,

OR THE

PRESERVATION OF FAVOURED RACES IN THE STRUGGLE
FOR LIFE.

BY CHARLES DARWIN, M.A.,

LONDON:
JOHN MURRAY, ALBEMARLE STREET.

See "The Black Cloud" in Project Physics Reader 2.

Chapter 5 Where is the Earth? – The Greeks' Answers

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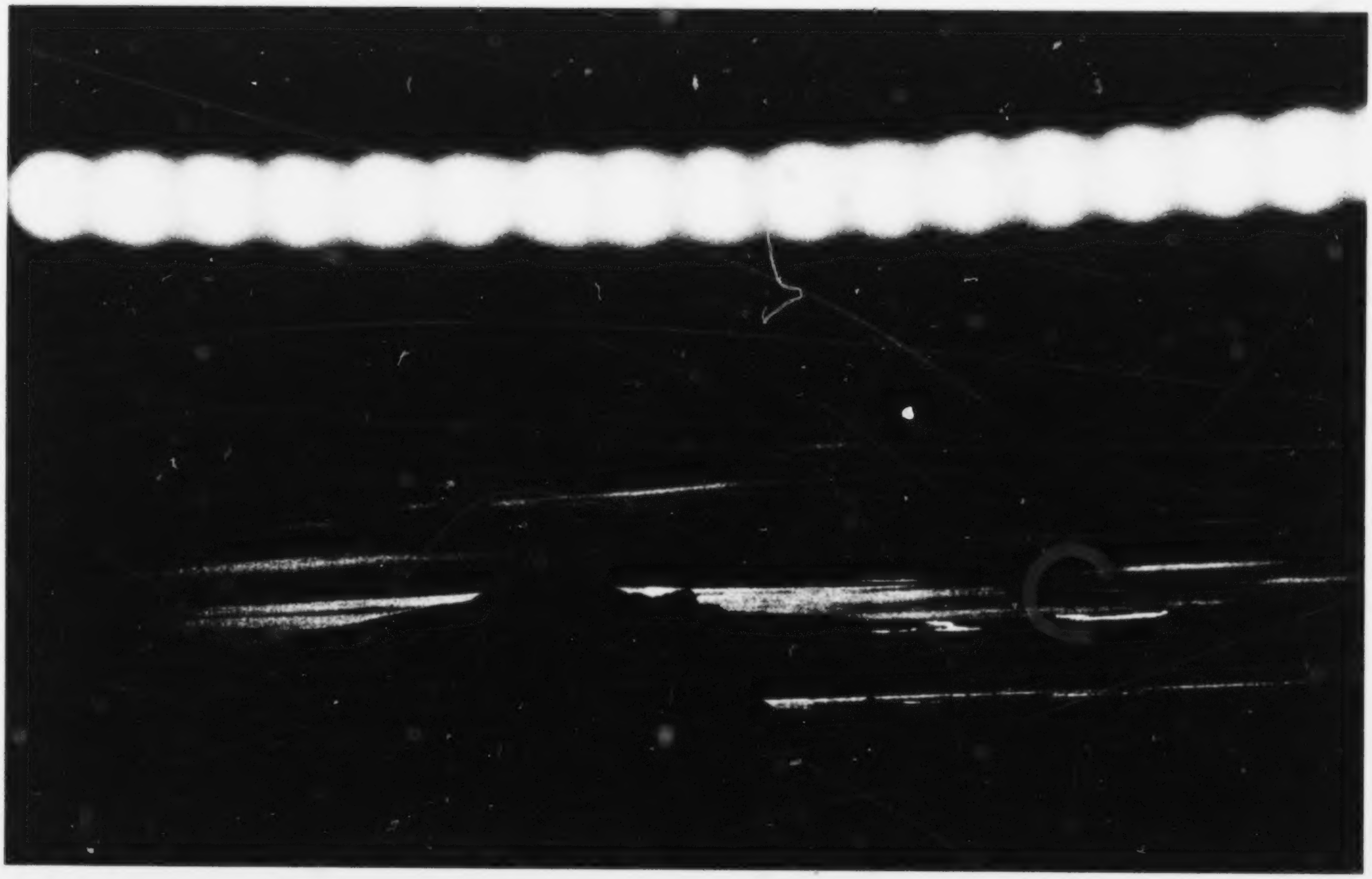


5.1 Motions of the sun and stars. The facts of everyday astronomy, the heavenly happenings themselves, are the same now as in the times of the Greeks. You can observe with the unaided eye most of what they saw and recorded. You can discover some of the long-known cycles and rhythms, such as the seasonal changes of the sun's height at noon and the monthly phases of the moon. If our purpose were only to make accurate forecasts of eclipses and planetary positions, we could, like the Babylonians and Egyptians, focus our attention on the cycles and rhythms. If, however, like the Greeks, we wished to explain these cycles, we must imagine some sort of relatively simple model or theory with which we can predict the observed variations. But before we can understand the several theories proposed in the past, we must review the major observations which the theories were to explain: the motions of the sun, moon, planets and stars.

Each day the sun rises above your horizon on the eastern side of the sky and sets on the western side. At noon, halfway between sunrise and sunset, the sun is highest above your horizon. Thus, a record of your observations of the sun would be similar to that shown in Fig. 5.1 (a). Every day the same type of pattern occurs from sunrise to sunset. Thus the sun, and indeed all the objects in the sky, show a daily motion. They rise on the eastern horizon, pass a highest point, and set on the western horizon. This is called the daily motion.

See "Roll Call" in Project Physics Reader 2.

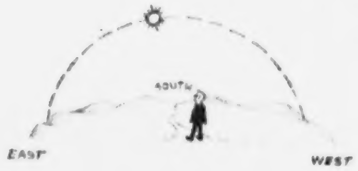
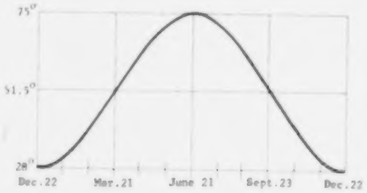


Fig. 5.1
a) Daily path of the sun through the sky.

Day by day from July through November, the noon height of the sun above the horizon becomes less. Near December 22, the sun's height at noon (as seen from the northern hemisphere) is least and the number of hours of daylight is smallest. During the next six months, from January into June, the sun's height at noon slowly increases. About June 21 it is greatest and the daylight is longest. Then the sun's gradual southward motion begins again, as Fig. 5.1 (b) indicates.



b) Noon altitude of the sun as seen from St. Louis, Missouri throughout the year.

This year-long change is the basis for the seasonal or solar year. Apparently the ancient Egyptians thought that the year had 360 days, but they later added five feast days to have a year of 365 days that fitted better with their observations of the seasons. Now we know that the solar year is 365.24220 days long. The fraction of a day, 0.24220, raises a problem for the calendar maker, who works with whole days. If you used a calendar of just 365 days, after four years New Year's Day would come early by one day. In a century you would be in error by almost a month. In a few centuries

Fig. 5.2 Midnight sun photographed at 5-minute intervals over the Ross Sea in Antarctica. The sun appears to move from right to left.

the date called January first would come in the summertime! In ancient times extra days or even whole months were inserted from time to time to keep a calendar of 365 days and the seasons in fair agreement.

SG 5.1

Such a makeshift arrangement, is, however, hardly satisfactory. In 45 B.C. Julius Caesar decreed a new calendar which averaged 365 1/4 days per year (the Julian calendar), with one whole day (a leap day) being inserted each fourth year. This calendar was used for centuries during which the small difference between the decimal parts 0.25 and 0.24220 accumulated to a number of days. Finally, in 1582 A.D., under Pope Gregory, a new calendar (the Gregorian calendar) was announced. This had only 97 leap days in 400 years. To reduce the number of leap days from 100 to 97 in 400 years, century years not divisible by 400 were omitted as leap years. Thus the year 1900 was not a leap year, but the year 2000 will be a leap year (try comparing 97/400 to 0.24220).

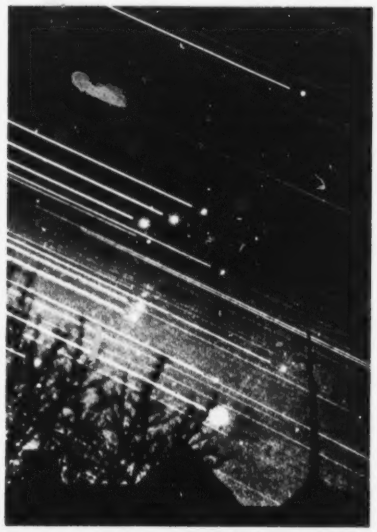


Fig. 5.3 Orion trails, a combination star and trail photograph. The camera shutter was opened for several hours while the stars trailed, then closed for a few minutes, then reopened while the camera was driven to follow the stars for a few minutes.

You may have noticed that a few stars are bright and many are faint. Some bright stars show colors, but most appear whitish. People have grouped many of the brighter stars into patterns, called constellations, such as the familiar Big Dipper and Orion. The brighter stars may seem to be larger, but if you look at them through binoculars, they still appear as points of light.

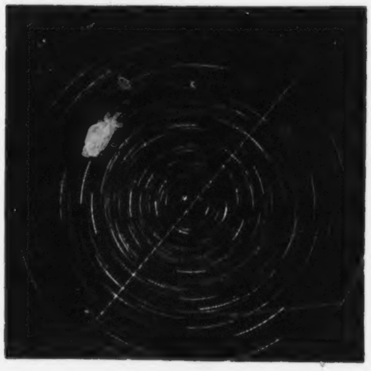
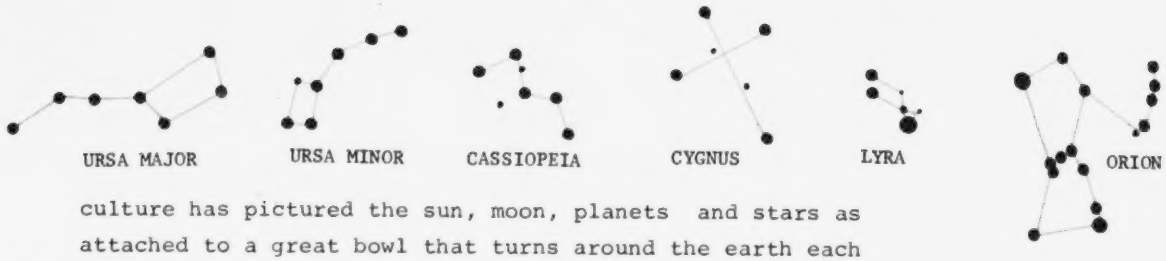


Fig. 5.4 Time exposure showing star trails around the North Celestial Pole. The diagonal line was caused by an artificial earth satellite. You can use a protractor to determine the duration of the exposure; the stars move about 15° per hour.

Which of the star trails in Fig. 5.4 might be that of Polaris? Defend your choice.

Have you noticed a particular star pattern overhead and then several hours later seen that it was near the western horizon? What happened? During the interval the stars in the western side of the sky moved down toward the horizon, while those in the eastern part of the sky moved up from the horizon. A photograph exposed for some time would show the trails of the stars, like those shown in Fig. 5.3. During the night, as seen from a point on the northern hemisphere of our earth, the stars appear to move counter-clockwise around a point in the sky called the North Celestial Pole, which is near the fairly bright star Polaris, as Fig. 5.4 suggests. Thus the stars like the sun show a daily motion across the sky.

Some of the star patterns, such as Orion (the Hunter) and Cygnus (the Swan, also called the Northern Cross), were named thousands of years ago. Since we still see the same star patterns described by the ancients, we can conclude that these star patterns change very little, if at all, over the centuries. Thus in the heavens we observe both stability over the centuries as well as smooth, orderly daily motion. To explain this daily rising and setting almost every early



culture has pictured the sun, moon, planets and stars as attached to a great bowl that turns around the earth each day.

But there are other motions in the sky. If you have observed the star patterns in the west just after twilight on two evenings several weeks apart, you have probably noticed during the second observation that the stars appeared nearer the horizon than when you made your first observation. As measured by sun-time, the stars set about four minutes earlier each day.

If you told time by the stars, would the sun set earlier or later each day?

From these observations we can conclude that the sun is slowly moving eastward relative to the stars, even though we cannot see the stars when the sun is above the horizon. One complete cycle of the sun against the background of stars takes a year. The sun's yearly path among the stars is called the ecliptic. It is a great circle in the sky and is tilted at about 23½° to the equator. The point at which the sun, moving along the ecliptic, crosses the equator from south to north on March 21 is called the Vernal Equinox. Thus we have three motions of the sun to explain: the daily rising and setting, the annual eastward cycle among the stars, and the north-south seasonal motion.

See "The Garden of Epicurus" in Project Physics Reader 2.

SG 5.2

Q1 What evidence do we have that the ancient people observed the sky?

Q2 For what practical purposes were calendars needed?

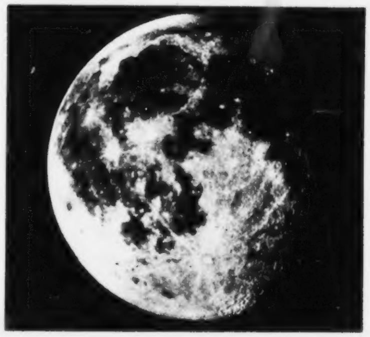
Q3 What are the observed motions of the sun during one year?

Q4 In how many years will the Gregorian calendar be off by one day?

5.2 Motions of the moon. The moon also moves eastward against the background of the stars and rises nearly an hour later each night. When the moon rises at sunset—when it is opposite the sun in the sky—the moon is bright and shows a full disc (full moon). About fourteen days later, when the moon is passing near the sun in the sky (new moon), we cannot see the moon at all. After new moon we first see the moon as a thin crescent low in the western sky at sunset. As the moon rapidly moves further eastward from the sun, the moon's crescent fattens to a half circle, called "quarter moon," and then within another week on to full moon again. After full moon, the pattern is reversed, with the moon slimming down to a crescent seen just before sunrise and then several days being invisible in the glare of sunlight.

These end of section questions are intended to help you check your understanding before going to the next section. Answers start on page 123.

As early as 380 B.C. the Greek philosopher Plato recognized that the phases of the moon could be explained by thinking of



Moon: 17 days old

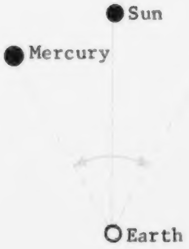
Moon: 3 days old

the moon as a globe reflecting sunlight and moving around the earth, with about 29 days between new moons. Because the moon appears to move so rapidly relative to the stars, people early assumed the moon to be close to the earth.

The moon's path around the sky is close to the yearly path of the sun; that is, the moon is always near the ecliptic. The moon's path is tipped a bit with respect to the sun's path; if it were otherwise, we would have an eclipse of the moon at every full moon and an eclipse of the sun at every new moon. The moon's motion is far from simple and has posed persistent problems for astronomers as, for example, predicting accurately the times of eclipses.

Q5 Describe the motion of the moon during one month.

Q6 Why don't eclipses occur each month?



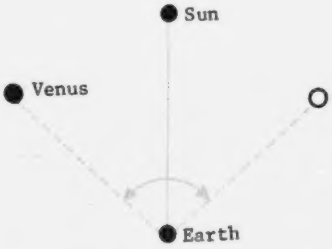


Fig. 5.5 The maximum angles from the sun at which we observe Mercury and Venus. Both planets can, at times, be observed at sunset or at sunrise. Mercury is never observed to be more than 28° from the sun, and Venus is never more than 48° from the sun.

5.3 The wandering stars. Without a telescope we can see, in addition to the sun and moon, five rather bright objects which move among the stars. These are the wanderers, or planets: Mercury, Venus, Mars, Jupiter and Saturn. (With the aid of telescopes three more planets have been discovered: Uranus, Neptune and Pluto; but none of these was known until nearly a century after the time of Isaac Newton.) Like the sun and moon, all the planets are observed to rise daily in the east and set in the west. Also like the sun and moon, the planets generally move eastward among the stars. But at certain times each planet stops moving eastward among the stars and for some months moves westward. This westward, or wrong-way motion, is called retrograde motion.

In the sky Mercury and Venus are always near the sun. As Fig. 5.5 indicates, the greatest angular distance from the sun is 28° for Mercury and 48° for Venus. Mercury and Venus show retrograde motion after they have been farthest east of the sun and visible in the evening sky. Then they quickly move westward toward the sun, pass it, and reappear in the morning sky. During this motion they are moving westward relative to the stars, as is shown by the plot for Mercury in Fig. 5.7.

In contrast, Mars, Jupiter and Saturn may be found in any position in the sky relative to the sun. As these planets (and the three discovered with the aid of telescopes) move eastward they pass through the part of the sky opposite to the sun. When they pass through the point 180° from the sun, i.e., when they are opposite to the sun, as Fig. 5.6 indicates, they are said to be in opposition. When each of these planets nears the time of its opposition, it ceases its eastward motion and for several months moves westward (see Fig. 5.7). As Table 5.1 and Fig. 5.7 show, the retrograde motion of Saturn lasts longer, and has a smaller angular displacement than do the retrograde motions of Jupiter and Mars.

Planet
Earth
Sun

Fig. 5.6 Opposition of a planet occurs when, as seen from the earth, the planet is opposite to the sun, and crosses the meridian at midnight.

Table 5.1 Recent Retrograde Motions of the Planets

Planet	Duration of Retrograde	Days*	Westward Displacement*
Mercury	April 26 to May 30, 1963	34	15°
Venus	May 29 to July 11, 1964	43	19°
Mars	Jan. 29 to April 21, 1965	83	22°
Jupiter	Aug. 10 to Dec. 6, 1963	118	10°
Saturn	June 4 to Oct. 21, 1963	139	7°
Uranus	Dec. 25, '65 to May 24, 1966	152	4°
Neptune	Feb. 22 to Aug. 1, 1966	160	3°
Pluto	Dec. 28, '65 to June 2, 1966	156	2°

*These intervals and displacements vary somewhat from cycle to cycle.

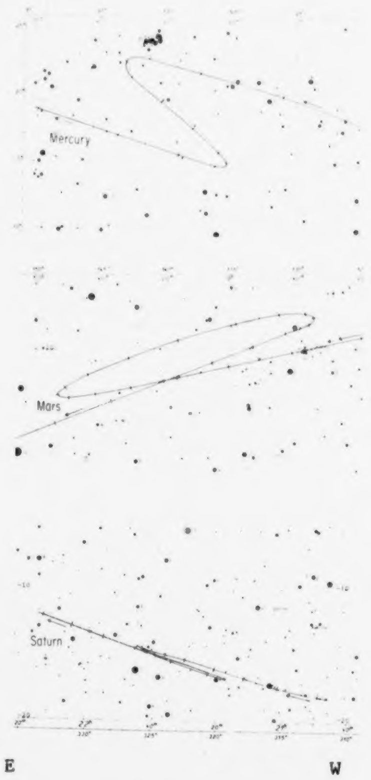


Fig. 5.7 The retrograde motions of Mercury (marked at 5-day intervals), Mars (at 10-day intervals), and Saturn (at 20-day intervals) in 1963, plotted on a star chart. The dotted line is the annual path of the sun, called the ecliptic.

The planets change considerably in brightness. When Venus is first seen in the evening sky as the "evening star," the planet is only fairly bright. But during the following four to five months as Venus moves farther eastward from the sun, Venus gradually becomes so bright that it can often be seen in the daytime if the air is clear. Later, when Venus scoots westward toward the sun, it fades rapidly, passes the sun, and soon reappears in the morning sky before sunrise as the "morning star." Then it goes through the same pattern of brightness changes, but in the opposite order: soon bright, then gradually fading. The variations of Mercury follow much the same pattern. But because Mercury is always seen near the sun during twilight, Mercury's changes are difficult to observe.

Mars, Jupiter and Saturn are brightest about the time that they are highest at midnight and opposite to the sun. Yet over many years their maximum brightness differs. The change is most noticeable for Mars; the planet is brightest when it is opposite the sun during August or September.

Not only do the sun, moon and planets generally move eastward among the stars, but the moon and planets (except Pluto) are always found within a band only 8° wide on either side of the sun's path.

Q7 In what part of the sky must you look to see the planets Mercury and Venus?

Q8 In what part of the sky would you look to see a planet which is in opposition?

Q9 When do Mercury and Venus show retrograde motion?

Q10 When do Mars, Jupiter and Saturn show retrograde motion?

Q11 In what ways do the retrograde motions of the planets differ? Is each the same at every cycle?

5.4 Plato's problem. About 400 B.C. Greek philosophers asked a new question: how can we explain the cyclic changes observed in the sky? Plato asked for a theory or general explanation to account for what was seen, or as he phrased it: "to save the appearances." The Greeks appear to have been the first people to desire theoretical explanations to account for natural phenomena. Their start was an important step toward science as we know it today.

How did the Greeks begin their explanation of the motions observed in the heavens? What were their assumptions?

At best, any answers to these questions must be tentative. While many scholars over the centuries have devoted themselves to the study of Greek thought, the documents available as the basis for our knowledge of the Greeks are mostly copies of copies and translations of translations in which errors and omissions occur. In some cases all we have are reports from later writers of what certain philosophers did or said, and these may be distorted or incomplete. The historians' task is difficult. Most of the original Greek writings were on papyrus or cloth scrolls, which have decayed through the ages. Many wars, much plundering and burning, have also destroyed many important documents. Especially tragic was the burning of the famous library of Alexandria in Egypt, which contained several hundred thousand documents. (Actually, it was burned three times: in part by Caesar's troops in 47 B.C.; then in 390 A.D. by a Christian fanatic; and the third time in 640 A.D. by Moslems when they overran the country.) Thus, while the general picture of Greek culture seems to be rather well established, many interesting particulars and details are not known.

The approach taken by the Greeks and their intellectual followers for many centuries was stated by Plato in the fourth century B.C. He stated the problem to his students in this way: the stars—eternal, divine, unchanging beings—move uniformly around the earth, as we observe, in that most perfect of all paths, the endless circle. But a few celestial objects, namely the sun, moon and planets, wander across the sky and trace out complex paths, including even retrograde motions. Yet, surely, being also heavenly bodies, they too must really move in a way that becomes their exalted status. This must be in some combination of circles. How then can we account for the observations of planetary motions and "save the appearances"? In particular, how can we explain the retrograde motions of the planets? Translated into more modern terms, the problem is: determine the combination of simultaneous uniform circular motions that must be assumed for each of the objects to account for the observed irregular motions. The phrase "uniform circular motion" means that the body moves around a center at a constant distance, and that the rate of angular motion around the center (such as one degree per day) is uniform or constant.

Notice that the problem is concerned only with the positions of the sun, moon, and planets. The planets appear to be only points of light moving against the background of stars. From two observations at different times we obtain a rate of motion: a value of so many degrees per day. The problem then is to find a "mechanism," some combination of motions, that will reproduce the observed angular motions and lead to predictions in agreement with observations. The ancient astronomers had no observational evidence about the distances of the planets; all they had were directions, dates and rates of angular motion. Although changes in brightness of the planets were known to be related to their positions relative to the sun, these changes in brightness were not included in Plato's problem.

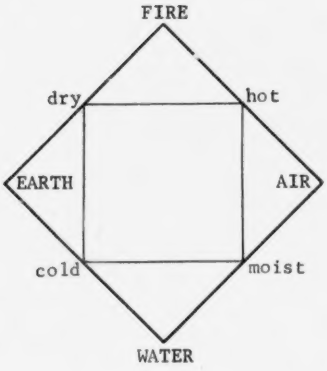
Plato's statement of this historic problem of planetary motion illustrates the two main contributions of Greek philosophers to our understanding of the nature of physical theories:

1. According to the Greek view, a theory should be based on self-evident concepts. Plato regarded as self-evident the concept that heavenly bodies must have perfectly circular motions. Only in recent centuries have we come to understand that such commonsense beliefs may be misleading. More than that—we have learned that every assumption must be critically examined and should be accepted only tentatively. As we shall often see in this course, the identification of hidden assumptions in science has been extremely difficult. Yet in many cases, when the assumptions have been identified and questioned, entirely new theoretical approaches have followed.

2. Physical theory is built on observable and measurable phenomena, such as the motions of the planets. Furthermore, our purpose in making a theory is to discover the uniformity of behavior, the hidden likenesses underlying apparent irregularities. For organizing our observations the language of number and geometry is useful. This use of mathematics, which is widely accepted today, was derived in part from the Pythagoreans, a group of mathematicians who lived in southern Italy about 500 B.C. and believed that "all things are numbers." Actually, Plato used the fundamental role of

numbers only in his astronomy, while Aristotle avoided measurements. This is unfortunate because, as the Prologue reported, the arguments of Aristotle were adopted by Thomas Aquinas, whose philosophy did not include the idea of measurement of change as a tool of knowledge.

Plato's assumption that heavenly bodies move with "uniform motion in perfect circles" was accepted for many centuries by those working on the problem of planetary motion. Not until around 1600 A.D. was this assumption abandoned.



Plato and many other Greek philosophers assumed that there were a few basic elements that mixed together to cause the apparent variety observed in the world. Although not everyone agreed as to what these elements were, gradually four were accepted as the explanation of phenomena taking place on earth. These elements were: Fire, Air, Water and Earth. Because substances found on earth contained various mixtures of these elements, these compound substances would have a wide range of properties and changes.

In the heavens, which were separate from the earth and were the abode of the gods, perfection must exist. Therefore motions in the heavens must be eternal and perfect, and the only perfect unending geometrical form was the circle. Also, the unchanging heavenly objects could not be composed of elements normally found at or near the earth, but were composed of a changeless fifth element of their own—the quintessence.

Plato's astronomical problem remained the most significant problem for theoretical astronomers for nearly two thousand years. To appreciate the later efforts and consequences of the different interpretation developed by Kepler, Galileo and Newton, let us examine what solutions to Plato's problem were developed by the Greeks.

Q12 What assumptions did Plato make in his problem?

Q13 Why is our knowledge of Greek science incomplete?

Q14 What basic assumption did the Greeks make about the nature of a theory?

5.5 The first earth-centered solution. The Greeks observed that the earth was obviously large, solid and permanent, while the heavens seemed to be populated by small, remote, ethereal objects that continuously went through their various motions. What was more natural than to conclude that our big, heavy earth was the steady, unmoving center of the universe? Such an earth-centered model is called geocentric. With it the daily motion of the stars could easily be explained: they were attached to, or were holes in, a large surrounding spherical dome, and were all at the same distance from us.

Daily, this celestial sphere would turn around an axis through the earth. As a result all the stars on it would move in circular paths around the pole of rotation. In this way the daily motions could be explained.

The observed motion of the sun through the year was explained by use of a more complex model. To explain the sun's motion among the stars a separate invisible shell was needed that carried the sun around the earth. To explain the observed annual north-south motion of the sun the axis of this sphere for the sun should be tipped from the axis of the dome of the stars. (See Fig. 5.8.)

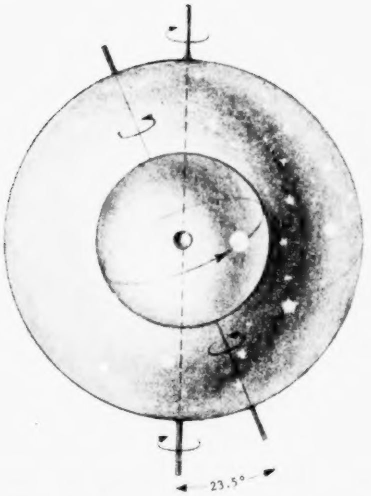
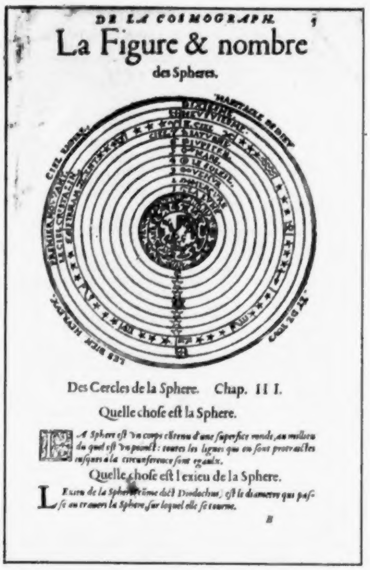


Fig. 5.8 The annual north-south (seasonal) motion of the sun was explained by having the sun on a sphere whose axis was tilted 23° from the axis of the sphere of the stars.

The motions of the planets—Mercury, Venus, Mars, Jupiter and Saturn—were more difficult to explain. Because Saturn moves most slowly among the stars, its sphere was assumed to be closest to the stars. Inside the sphere for Saturn would be spheres for faster-moving Jupiter and Mars. Since they all require more than a year for one trip around the sky, these three planets were believed to be beyond the sphere of the sun. Venus, Mercury and the moon were placed between the sun and the earth. The fast-moving moon was assumed to reflect sunlight and to be closest to the earth.

Such an imaginary system of transparent shells or spheres can provide a rough "machine" to account for the general motions of heavenly objects. By choosing the sizes, rates and directions of motion of the supposed linkages between the various spheres a rough match could be made between the model and the observations (as in Fig. 5.9). If additional observations reveal other cyclic variations, more spheres and linkages can be added.

Eudoxus, Plato's friend, concluded that 27 spheres or motions would account for the general pattern of motions. Later Aristotle added 29 more motions to make a total of 56. An interesting description of this system is given by the poet Dante in the Divine Comedy, written in 1300 A.D., shortly after Aristotle's writings became known in Europe.



DE LA COSMOGRAPH. 5

La Figure & nombre des Spheres.

Des Cercles de la Sphere. Chap. III.

Quelle choſe eſt la Sphere.

La Sphere eſt vn corps cõtenu d'une ſuperfice ronde, au milieu du quel eſt vn poinct: toutes les lignes qui en ſont protraictes iuſques a la circunference ſont egaulx.

Quelle choſe eſt l'exieu de la Sphere.

L'Exieu de la Sphere, cõme dict Diodochus, eſt le diametre qui paſſe au trauers la Sphere ſur lequel elle ſe tourne.

B

Fig. 5.9 A geocentric cosmological scheme. The earth is fixed at the center of concentric rotating spheres. The sphere of the moon (lune) separates the terrestrial region (composed of concentric shells of the four elements Earth, Water, Air and Fire) from the celestial region. In the latter are the concentric spheres carrying Mercury, Venus, Sun, Mars, Jupiter, Saturn and the stars. To simplify the diagram, only one sphere is shown for each planet. (From the DeGolyer copy of Petrus Apianus' Cosmographie, 1551.)

Aristotle was not happy with this system, for it did not get the heavenly bodies to their observed positions at quite the right times. In addition, it did not account at all for the observed variations in brightness of the planets. But we must not ridicule Greek science for being different from our science. The Greeks were just beginning the development of scientific theories and inevitably made assumptions that we now consider unsuitable. Their science was not "bad science," but in many ways it was a quite different kind of science from ours. Furthermore, we must realize that to scientists 2000 years from now our efforts may seem strange and inept.

Even today scientific theory does not and cannot account for every detail of each specific case. As you have already seen, important general concepts, like velocity and acceleration, must be invented for use in organizing the observations. Scientific concepts are idealizations which treat selected aspects of observations rather than the totality of the raw data.

SG 5.3

As you might expect, the history of science contains many examples in which the aspects neglected by one researcher turned out later to be quite important. But how would better systems for making predictions be developed unless there were first trials? Through tests and revisions theories may be improved or they may be completely replaced.

Q15 What is a geocentric system?

Q16 Describe the first solution to Plato's problem.

5.6 A sun-centered solution. For nearly two thousand years after Plato and Aristotle the geocentric model was generally accepted. However, another radically different model, based on different assumptions, had been proposed. In the third century B.C., Aristarchus, perhaps influenced by the writings of Heracleides, who lived a century earlier, suggested that a much simpler description of heavenly motion would result if the sun were considered to be at the center, with the earth, planets and stars all revolving around it. A sun-centered system is called heliocentric.

Unfortunately, because the major writings of Aristarchus have been lost, our knowledge of his work is based mainly on comments made by other writers. Archimedes wrote that Aristarchus found, from a long geometrical analysis, that the sun must be at least eighteen times farther away than the moon. Since the distance to the moon was known roughly, this put the sun several million miles away. Furthermore Aristarchus concluded that this distant sun must be much larger than the earth. He believed that the larger body, which was also the source of sunlight, should be at the center of the universe.

Aristarchus proposed that all the daily motions observed in the sky could be easily explained by assuming that the earth rotates daily on an axis. Furthermore, the annual changes in the sky, including the retrograde motions of the planets, could be explained by assuming that the earth and the five visible planets move around the sun. In this model the motion previously assigned to the sun around the earth was assigned to the earth moving around the sun. Also, notice that the earth became just one among several planets.

How such a system can account for the retrograde motions of Mars, Jupiter and Saturn can be seen from Figs 5.10 (a) and (b), in which an outer planet and the earth are assumed to be moving around the sun in circular orbits. The outer planet is moving more slowly than the earth. As a result, when we see the planet nearly opposite to the sun, the earth moves rapidly past the planet, and to us the planet appears to be moving backward, that is westward, or in retrograde motion, across the sky.

The heliocentric (sun-centered) hypothesis has one further advantage. It explains the bothersome observation that the planets are brighter and presumably nearer the earth during their retrograde motion.

Even so, the proposal by Aristarchus was attacked on three bases:

First, it did violence to the philosophical doctrines that the earth, by its very immobility and position, is different from the celestial bodies, and that the natural place of the earth is the center of the universe. In fact, his contemporaries considered Aristarchus impious for suggesting that the earth moved. Also, the new picture of the solar system contradicted common sense and everyday observations: certainly, the earth seemed to be at rest.

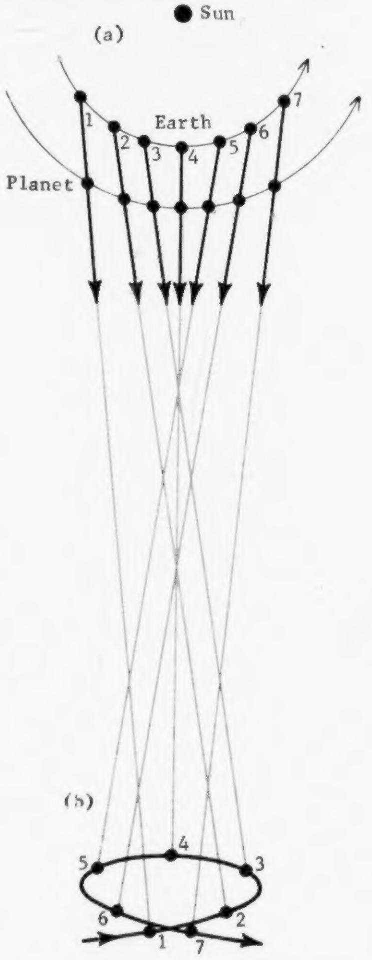
Second, the attackers offered observational evidence to refute Aristarchus. If the earth were moving in an orbit around the sun, it would also be moving back and forth below the fixed stars, such as the North Star. Then the angle from the vertical at which we have to look for any star would be different when seen from the various points in the earth's annual path (see Fig. 5.11). This shift, called the parallactic shift of the fixed stars, should occur on the basis of Aristarchus' heliocentric hypothesis. But it was not observed by the Greek astronomers.

This awkward fact could be explained in two ways. The stellar parallax could be too small to be observed with the naked eye, though this would require the stars to be either enormously distant, perhaps some hundreds of millions of miles away. Or the earth could be fixed, and the theory of Aristarchus was wrong.

Today with telescopes we can observe the parallactic shift of stars so we know that Aristarchus was correct. The

Fig. 5.10 Retrograde motion of an outer planet. The numbers in (b) correspond to the numbered positions in (a).

(a) Actual configurations of sun, earth and planet for retrograde motion.



(b) Apparent path of the planet against the background stars as seen from the earth.

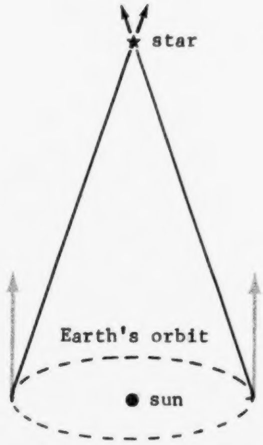


Fig. 5.11 How the changing position of the earth in its orbit should cause a parallactic shift in a star's position.

stellar parallax is too small to be seen with the naked eye—and indeed so small that even with telescopes it was not measured until 1838. The largest parallactic shift is an angle of only 1.5 seconds of arc, equivalent to the diameter of a penny seen at a distance of about two miles! The parallax exists, but we can sympathize with the Greeks who rejected the heliocentric theory because the parallactic shift required by the theory could not be observed at that time.

Third, Aristarchus does not seem to have used his system for making predictions of planetary positions. His work seems to have been purely qualitative, a general picture of how things might be.

Here were two different ways to describe the same observations. But the new proposal required a drastic change in man's image of the universe—not to speak of the fact that the stellar parallax which it predicted was not observed. Actually Aristarchus' heliocentric hypothesis had so little influence on Greek thought that we might have neglected it here as being unimportant. But fortunately his arguments were recorded and eighteen centuries later stimulated the thoughts of Copernicus. Ideas, it seems, are not bound by space or time, and cannot be accepted or dismissed with final certainty.

Q17 What two radically new assumptions were made by Aristarchus?

Q18 How can the model proposed by Aristarchus explain retrograde motion?

Q19 What change predicted by Aristarchus' theory was not observed by the Greeks?

Q20 Why was Aristarchus considered impious?

5.7 The geocentric system of Ptolemy. Disregarding the heliocentric model suggested by Aristarchus, the Greeks continued to develop their planetary theory as a geocentric system. As we have seen, the first solution in terms of crystalline spheres lacked accuracy. During the 500 years after Plato and Aristotle, astronomers began to want a more accurate theory for the heavenly timetables. Of particular importance were the positions of the sun, moon and planets along the ecliptic. A better theory must account for both the large general motions and the numerous smaller cyclic variations. To fit the observations, a complex theory was needed for each planet.

Several Greek astronomers made important contributions which resulted about 150 A.D. in the geocentric theory of Claudius Ptolemy of Alexandria. Ptolemy's book on the motions of the heavenly objects is a masterpiece of analysis, which used many new geometrical solutions.

Ptolemy wanted a system that would predict accurately the positions of each planet. But the type of system and motions he accepted was based on the assumptions of Aristotle. In the preface to The Almagest Ptolemy states:

> For indeed Aristotle quite properly divides also the theoretical [in contrast to the practical] into three immediate genera: the physical, the mathematical, and the theological....The kind of science which seeks after Him is the theological; for such an act can only be thought as high above somewhere near the loftiest things of the universe and is absolutely apart from sensible things. But the kind of science which traces through the material and the hot, the sweet, the soft, and such things, would be called physical, and such an essence... is to be found in corruptible things and below the the lunar sphere. And the kind of science which shows up quality with respect to forms and local motions, seeking figure, number, and magnitude, and also place, time, and similar things, would be defined as mathematical.

Then he defines the problem and states his assumptions:

> ...we wish to find the evident and certain appearances from the observations of the ancients and our own, and applying the consequences of these conceptions by means of geometrical demonstrations.
>
> And so, in general, we have to state, that the heavens are spherical and move spherically; that the earth, in figure, is sensibly spherical...; in position, lies right in the middle of the heavens, like a geometrical center; in magnitude and distance, [the earth] has the ratio of a point with respect to the sphere of the fixed stars, having itself no local motion at all.

Ptolemy then argues that each of these assumptions is necessary and fits with all our observations. The strength of his belief is illustrated by his statement: "...it is once for all clear from the very appearances that the earth is in the middle of the world and all weights move towards it." Notice that he has mixed the astronomical observations with the physics of falling bodies. This mixture of astronomy and physics became highly important when he referred to the proposal of Aristarchus that the earth might rotate and revolve:

> Now some people, although they have nothing to oppose to these arguments, agree on something, as they think, more plausible. And it seems to them there is nothing against their supposing, for instance, the heavens immobile and the earth as turning on the same axis [as the stars] from west to east very nearly one revolution a day....
>
> But it has escaped their notice that, indeed, as far as the appearances of the stars are concerned, nothing would perhaps keep things from

> being in accordance with this simpler conjecture, but that in the light of what happens around us in the air such a notion would seem altogether absurd.

Here Ptolemy recognizes that the "simple conjecture" of a moving and rotating earth would perhaps satisfy the astronomical observations. But he rejects this conjecture by spelling out what would "happen around us in the air." The earth would spin at a great speed under the air, with the result that all clouds would fly past toward the west, and all birds or other things in the air would be carried away to the west. If, however, the air turned with the earth, "none the less the bodies contained in it would always seem to be outstripped by the movement of both [the earth and the air]."

On these assumptions and arguments Ptolemy developed very clever and rather accurate procedures by which the positions of each planet could be derived on a geocentric model. In the solutions he used circles and three other geometrical devices. Each device provided for variations in the rate of angular motion as seen from the earth. To appreciate Ptolemy's problem, let us examine one of the many small variations he was attempting to explain.

One irregularity that must be explained will be immediately apparent if you consult a calendar. Let us divide the sun's total path around the sky into four equal segments, each an arc of 90°, and start from where the sun's path crosses the celestial equator on March 21. Although there may be a variation of one day between years, due to the introduction of a "leap day," the sun is usually farthest north on June 21, back at the equator on September 23, then farthest south on December 22.

As Table 5.2 shows, the actual motion of the sun during the year is not at a uniform rate.

Center a protractor on point C of Fig. 5.12 and measure the degrees in the arcs 1-2, 2-3, 3-4 and 4-1. Consider each 1° around C as one day. Make a graph of the days needed for the planet to move through the four arcs as seen from the earth, E.

Table 5.2

Irregular Motion of the Sun in Moving through 90° Arcs

	Day count of year	Difference in days
March 21	80th day	
		92
June 21	172nd day	
		94
September 23	266th day	
		90
December 22	356th day	
		89
March 21 (80 + 365)	445th day	

1. The eccentric. Previously, astronomers had held, with Plato, that the motion of a planet must be at a uniform angular rate and at a constant distance from the center of the earth. This is what we defined in Sec. 5.4 as uniform circular motion. Although Ptolemy believed that the earth was at the center of the universe, it need not be at the center around which the radius turned at a uniform rate. He used a new arrangement called an eccentric (Fig. 5.12) which has the radius of constant length moving uniformly around the center C, but with the earth E located off-center. As seen from the off-center earth, the planets, sun, etc., would require unequal numbers of days to move through the quadrants, 1-2, 2-3, etc.

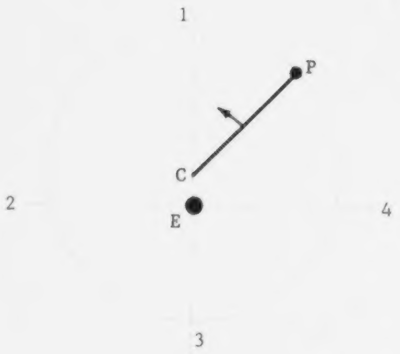


Fig. 5.12 An eccentric. The angular motion is at a uniform rate around the center, C. But the earth, E, is off-center.

An eccentric motion, as shown in Fig. 5.12, will account for the type of irregularity reported in Table 5.2. However, the scale of Fig. 5.12 is misleading; the earth need be offset from the center by only a small amount to satisfy the data of Table 5.2. Notice that Ptolemy was giving up the old notion that the earth must be at the center of the motion.

2. The epicycle. While the eccentric can account for small variations in the rate of motion, it cannot describe any such radical change as retrograde motion. To account for retrograde motion, Ptolemy used another type of motion, the epicycle (see Fig. 5.13). The planet P is considered to be moving at a uniform rate on the circumference of a small circle, called the epicycle. The center of the epicycle D moves at a uniform rate on a large circle, called the deferent, around its center C.

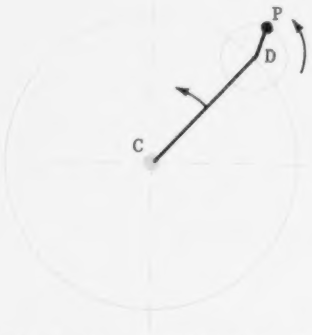


Fig. 5.13 An epicycle. The planet P revolves on its epicycle about D. D revolves on the deferent (large circle) centered at the Earth C.

With a relatively large radius or short period for the epicycle, the planet would be seen to move through loops. If from the center C we look out nearly in the plane of the motion, these loops would look like retrograde motions. Fig. 5.14 shows the motions produced by a simple mechanical model, an "epicycle machine."

An epicycle can be used to describe many kinds of motion. We may select the ratio of the radius of the epicycle to that of the deferent. Also we may choose the directions and rates of angular motion of the epicycle. To obtain apparent retrograde motion as seen from the center of the deferent, the epicycle must turn rapidly or have a radius which is a sizable fraction of the radius of the deferent.

To describe the three outer planets Ptolemy had to make a strange assumption. As Fig. 5.15 shows, he had to have the radius of each epicycle always parallel to the line from

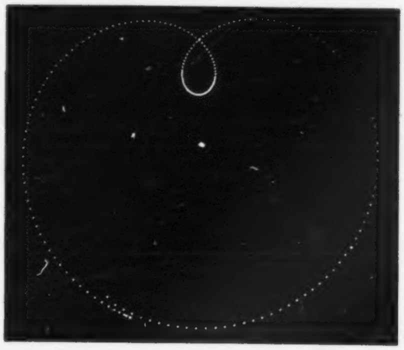
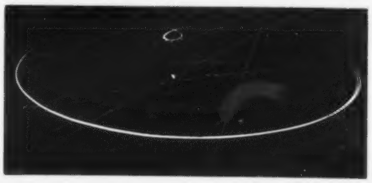

(a)

Fig. 5.14 Retrograde motion created by a simple epicycle machine.
(a) Stroboscopic photograph of epicyclic motion. The flashes were made at equal time intervals. Note that the motion is slowest in the loop.
(b) Loop seen from near its plane.


(b)

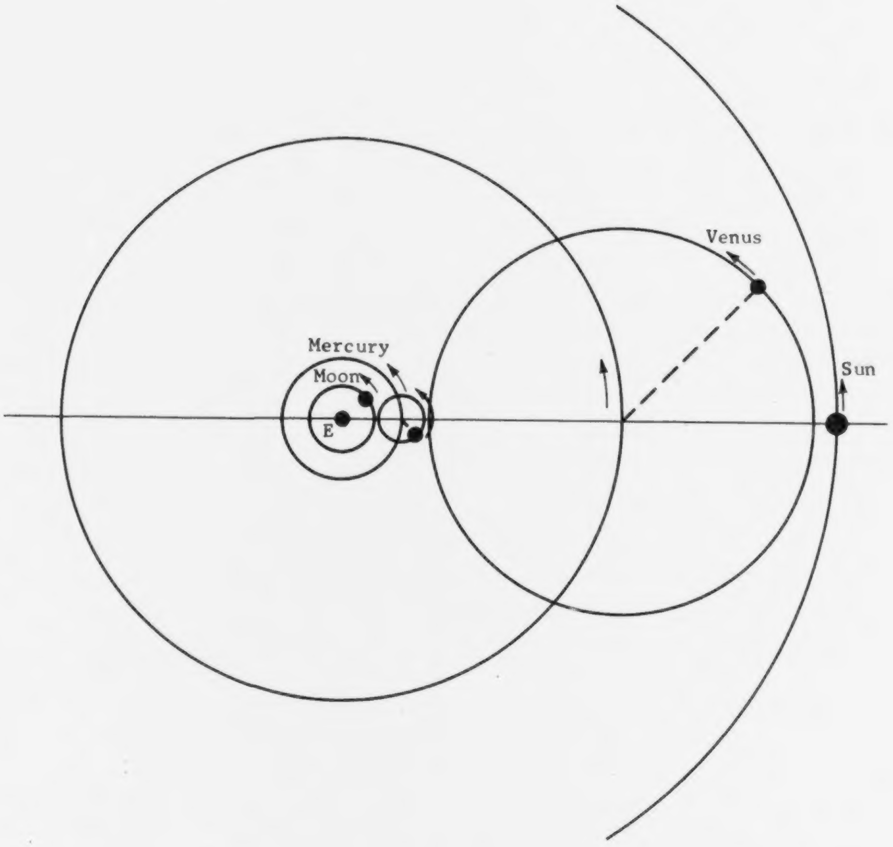
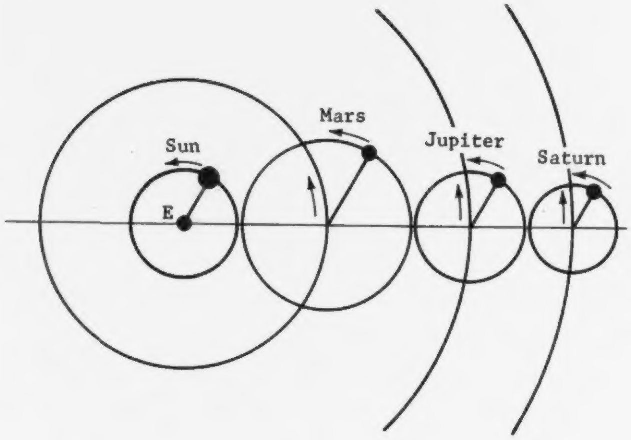

Venus
Mercury
Moon
Sun
E

Fig. 5.15 Simplified representation of the Ptolemaic system. The scale of the upper drawing, which shows the earth and sun, is ten times that of the lower drawing, which shows the planets that are further than the sun. The planets are shown along a line to emphasize the relative sizes of the epicycles. The epicycles of the moon are not included.


Sun
E
Mars
Jupiter
Saturn

the sun to the earth. This required that each epicycle have a period of exactly one year. As we shall see in Ch. 6, Copernicus wondered about this limitation of the epicycles.

Not only may the use of an epicycle describe retrograde motion, it also could explain the greater brightness of the planets when they were near opposition, as you can see from Fig. 5.14. However, to accept this explanation of the changes in brightness would oblige us to assume that the planets actually moved through space on epicycles and deferents. This assumption of "real motions" is quite different from that of considering the epicycles and deferents as only useful computing devices, like algebraic equations.

3. The equant. But even with combinations of eccentrics and epicycles Ptolemy was not able to fit the motions of the five planets. There were more variations in the rates at which the planets moved than could be fitted by eccentrics and epicycles. For example, as we see in Fig. 5.16, the retrograde motion of Mars is not always of the same angular size or duration. To allow for these motions Ptolemy introduced a third geometrical device, called the equant (Fig. 5.17), which is a modified eccentric. The uniform angular motion is around an off-center point C, while the earth E (and the observer) is equally off-center, but in the opposite direction.

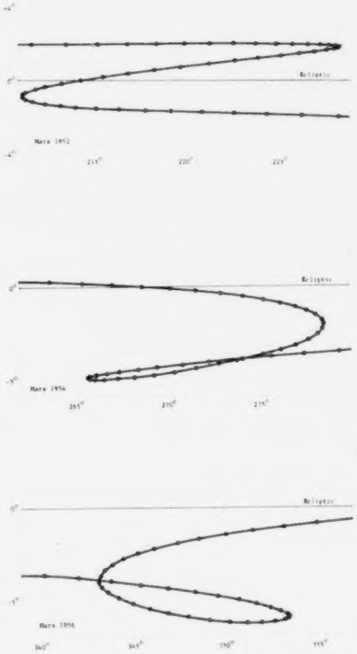


Fig. 5.16 Mars is plotted at four-day intervals on three consecutive oppositions. Note the different sizes and shapes of the retrograde curves.

Although Ptolemy displaced the earth from the center of the motion, he always used a uniform rate of angular motion around some center. To that extent he stayed close to the assumptions of Plato. By a combination of eccentrics, epicycles, and equants he described the motion of each planet separately. His geometrical analyses were equivalent to a complicated equation for each individual planet. Ptolemy did not picture these motions as an interlocking machine where each planet moved the next. However, Ptolemy adopted the old order of distances: stars, Saturn, Jupiter, Mars, Sun, Venus, Mercury, Moon, Earth. Because there was no information about the distances of the planets, the orbits are usually shown nested inside each other so that their epicycles do not quite overlap (see Fig. 5.15).

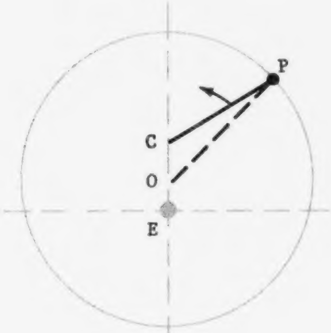


Fig. 5.17 An equant. The planet P moves at a uniform rate around the off-center point C. The earth E is equally off-center in the opposite direction.

Notice how radically this set of geometrical motions differed from the propositions of Plato stated 500 years earlier. Although Ptolemy used uniform angular motions and circles, the centers and radii of these motions could now be adjusted and combined to provide the best fit with observations. No longer was the center of the motion at the earth, but the center could be offset by whatever amount was needed. For each planet separately Ptolemy had a combination of motions that predicted its observed positions over long periods of time to within about two degrees.

However, there were some difficulties. For example, his proposed motions for the moon involved such large epicycles that during a month the observed angular diameter of the moon should change by a factor of two.

The Ptolemaic description was a series of mathematical devices to match and predict the motion of each planet separately. A recently discovered manuscript of Ptolemy's describes his picture of how the planet orbits were related in a way similar to that shown in Fig. 5.15. Nevertheless, in the following centuries most people—including Dante—believed that the planets really moved on some sort of crystalline spheres as Eudoxus had suggested, but that the motions were described mathematically by Ptolemy's combinations of geometric devices.

In Ptolemy's theory of the planetary motions there were, as in all theories, a number of assumptions:

1. that the heaven is spherical in form and rotates around the earth once a day;
2. that the earth is spherical;
3. that the earth is at the center of the heavenly sphere;
4. that the size of the earth is negligible compared to the distance to the stars;
5. that the earth has no motions;
6. that uniform angular motion along circles is the only proper behavior for celestial objects.

Although now discarded, the Ptolemaic system, proposed in 150 A.D., was used for about 1500 years. What were the major reasons for this long acceptance? Ptolemy's theory:

1. predicted fairly accurately the positions of the sun, moon and planets;
2. did not predict that the fixed stars should show a parallactic shift;
3. agreed in most details with the philosophical doctrines developed by the earlier Greeks, including the idea of "natural motion" and "natural place";
4. had commonsense appeal to all who saw the sun, moon, planets and stars moving around them;
5. agreed with the comforting assumption that we live on an immovable earth at the center of the universe.

Also:

6. could survive better because at that time there were very few theoretical astronomers.
7. fitted in with Thomas Aquinas' widely accepted synthesis of Christian belief and Aristotelian physics.

SG 5.4

Yet Ptolemy's theory was eventually displaced by a heliocentric one. Why did this occur? What advantages did the new theory have over the old? From our present point of view, on what basis do we say that a scientific theory is successful or unsuccessful? We shall have to face such persistent questions in what follows.

Q21 What assumptions did Ptolemy make for his theory?

Q22 What arguments did Ptolemy use against the idea that the earth rotated?

Q23 What limitation did Ptolemy have to assign to his epicycles?

Q24 Was the Ptolemaic system proposed as a description of real planetary orbits, or only as a means for computing positions?

Q25 In what way did Ptolemy disregard the assumptions of Plato?

Q26 Why was Ptolemy's system accepted for more than a thousand years?

Study Guide

5.1 a) List the observations of the motions of heavenly bodies that you might make which would have been possible in ancient Greek times.

b) For each observation, list some reasons why the Greeks thought these motions were important.

5.2 Describe the apparent motions of the stars and their times of rising and setting if the earth's shape were:

a) saucer-shaped,
b) flat,
c) a pyramid or cube,
d) a cylinder having its axis north-south.

5.3 Throughout Chapter 5, many references are made to the importance of recording observations accurately.

a) Why is this so important in astronomy?
b) Why are such records more important in astronomy than in other areas of physics you have already studied?

5.4 As far as the Greeks were concerned, and indeed as far as we are concerned, a reasonable argument can be made for either the geocentric or the heliocentric theory of the universe.

a) In what ways were both ideas successful?
b) In terms of Greek science, what are some advantages and disadvantages of each system?
c) What were the major contributions of Ptolemy?

Chapter 6 Does the Earth Move? – The Works of Copernicus and Tycho

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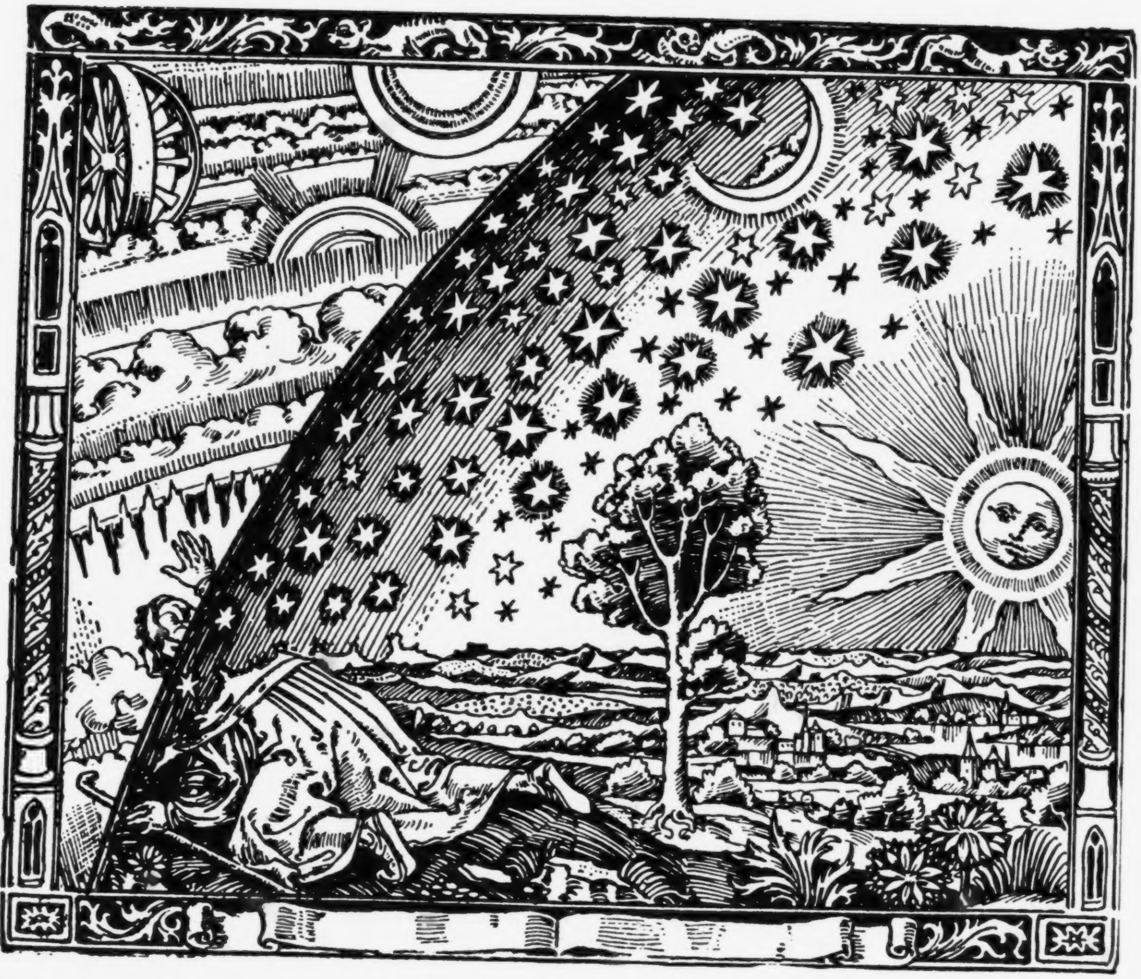


Man breaking through the vault of the heavens to new spheres. (Woodcut, circa 1530.)

6.1 The Copernican system. Nicolaus Copernicus (1473-1543) was still a young student in Poland when Columbus discovered America. Copernicus was an outstanding astronomer and mathematician, and also was a talented and respected churchman, jurist, administrator, diplomat, physician, classicist and economist. During his studies in Italy he learned Greek and read the writings of earlier philosophers and astronomers. As a canon of the Cathedral of Frauenberg he was busy with civic and church affairs, but increasingly he worked on his astronomical studies. On the day of his death in 1543, he saw the first copy of his great book which opened a whole new vision of the universe.
6.1).

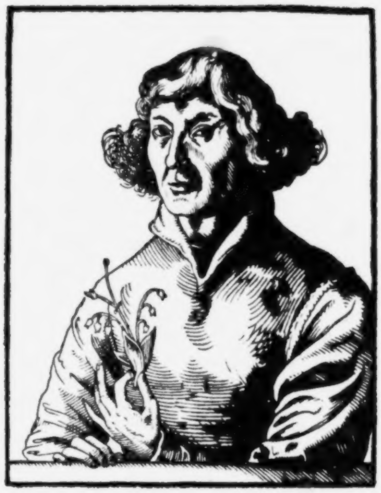


Fig. 6.1 Nicolaus Copernicus (1473-1543). In Polish his name was Kopernik.

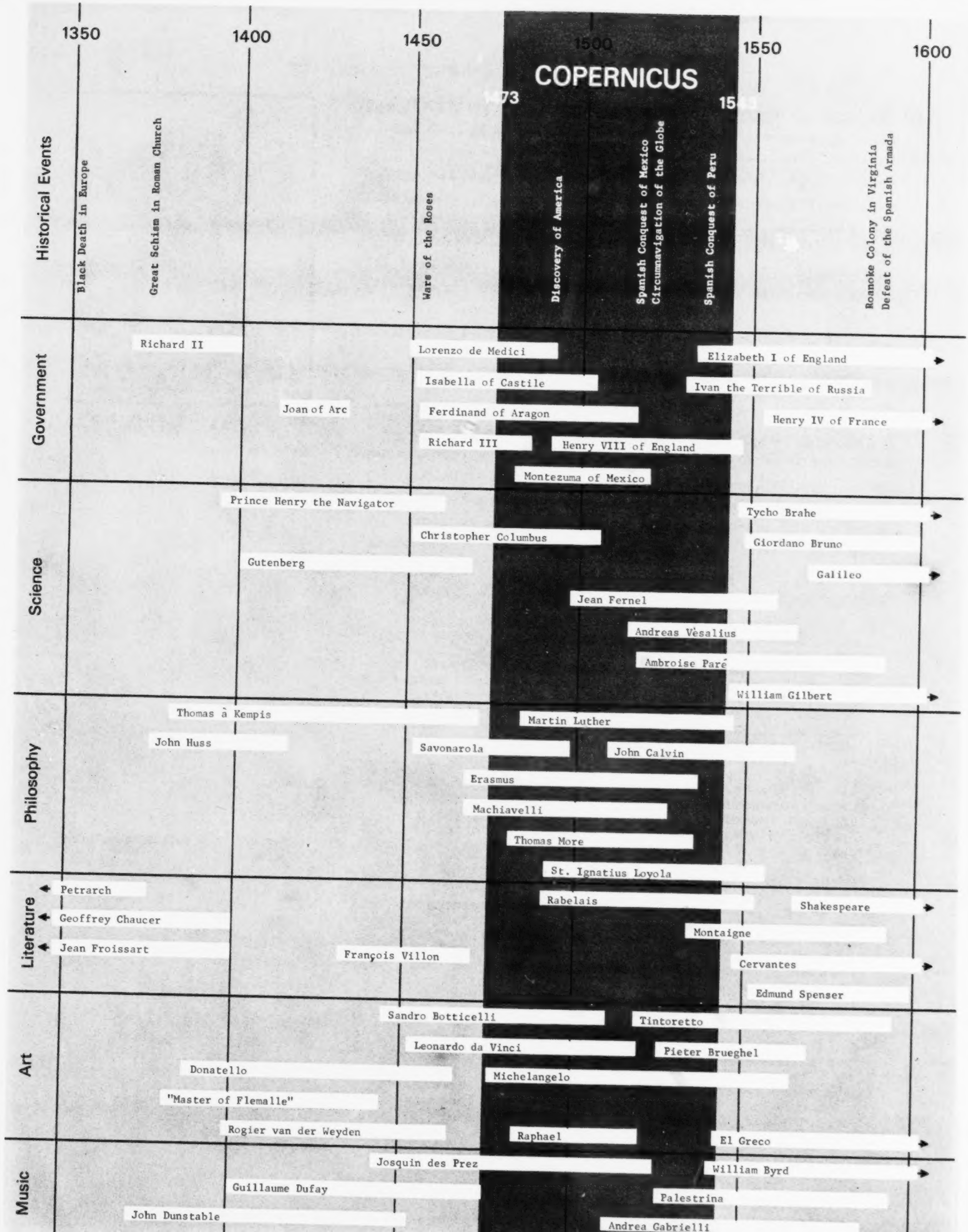
Copernicus titled his book De Revolutionibus Orbium Coelestium, or On the Revolutions of the Heavenly Spheres, which suggests an Aristotelian notion of concentric spheres. Copernicus was indeed concerned with the old problem of Plato: the construction of a planetary system by combinations of the fewest possible uniform circular motions. He began his study to rid the Ptolemaic system of the equants which were contrary to Plato's assumptions. In his words, taken from a short summary written about 1530,

> ...the planetary theories of Ptolemy and most other astronomers, although consistent with the numerical data, seemed likewise to present no small difficulty. For these theories were not adequate unless certain equants were also conceived; it then appeared that a planet moved with uniform velocity neither on its deferent nor about the center of its epicycle. Hence a system of this sort seemed neither sufficiently absolute nor sufficiently pleasing to the mind.
>
> Having become aware of these defects, I often considered whether there could perhaps be found a more reasonable arrangement of circles, from which every apparent inequality would be derived and in which everything would move uniformly about its proper center, as the rule of absolute motion requires.

What was "the rule of absolute motion"?

In De Revolutionibus he wrote:

> We must however confess that these movements [of the sun, moon, and planets] are circular or are composed of many circular movements, in that they maintain these irregularities in accordance with a constant law and with fixed periodic returns, and that could not take place, if they were not circular. For it is only the circle which can bring back what is past and over with....
>
> I found first in Cicero that Nicetas thought that the Earth moved. And afterwards I found in Plutarch that there were some others of the same opinion.... Therefore I also...began to meditate upon the mobility of the Earth. And although the opinion seemed absurd, nevertheless, because I knew that others before me had


1350
1400
1450
1500
1550
1600
COPERNICUS
73
15
Historical Events
Black Death in Europe
Great Schism in Roman Church
Wars of the Roses
Discovery of America
Spanish Conquest of Mexico
Circumnavigation of the Globe
Spanish Conquest of Peru
Roanoke Colony in Virginia
Defeat of the Spanish Armada
Government
Richard II
Lorenzo de Medici
Elizabeth I of England
Isabella of Castile
Ivan the Terrible of Russia
Joan of Arc
Ferdinand of Aragon
Henry IV of France
Richard III
Henry VIII of England
Montezuma of Mexico
Science
Prince Henry the Navigator
Tycho Brahe
Christopher Columbus
Giordano Bruno
Gutenberg
Galileo
Jean Fernel
Andreas Vesalius
Ambroise Paré
William Gilbert
Philosophy
Thomas à Kempis
Martin Luther
John Huss
Savonarola
John Calvin
Erasmus
Machiavelli
Thomas More
St. Ignatius Loyola
Literature
Petrarch
Rabelais
Geoffrey Chaucer
Shakespeare
Montaigne
Jean Froissart
François Villon
Cervantes
Edmund Spenser
Art
Sandro Botticelli
Tintoretto
Leonardo da Vinci
Pieter Brueghel
Donatello
Michelangelo
"Master of Flemalle"
Rogier van der Weyden
Raphael
El Greco
Music
Josquin des Prez
William Byrd
Guillaume Dufay
Palestrina
John Dunstable
Andrea Gabrielli

> been granted the liberty of constructing whatever circles they pleased in order to demonstrate astral phenomena, I thought that I too would be readily permitted to test whether or not, by the laying down that the Earth had some movement, demonstrations less shaky than those of my predecessors could be found for the revolutions of the celestial spheres....I finally discovered by the help of long and numerous observations that if the movements of the other wandering stars are correlated with the circular movement of the Earth, and if the movements are computed in accordance with the revolution of each planet, not only do all their phenomena follow from that but also this correlation binds together so closely the order and magnitudes of all the planets and of their spheres or orbital circles and the heavens themselves that nothing can be shifted around in any part of them without disrupting the remaining parts and the universe as a whole.

After nearly forty years of study Copernicus proposed a system of more than thirty eccentrics and epicycles which would "suffice to explain the entire structure of the universe and the entire ballet of the planets." Like the Almagest, De Revolutionibus uses long geometrical analyses and is difficult to read. Examination of the two books strongly suggests that Copernicus thought he was producing an improved version of the Almagest. He used many of Ptolemy's observations plus a few more recent ones. Yet his system, or theory, differed from that of Ptolemy in several fundamental ways. Like all scientists, Copernicus made many assumptions as the basis for his system:

See the Preface to Copernicus' Revolutionibus in Project Physics Reader 2.

"1. There is no one center of all the celestial circles or spheres.

2. The center of the earth is not the center of the universe, but only of [gravitation] and of the lunar sphere.

3. All the spheres revolve about the sun...and therefore the sun is the center of the universe.

4. The ratio of the earth's distance from the sun to the [sphere of the stars] is so much smaller than the ratio of the earth's radius to its distance from the sun that the distance from the earth to the sun is imperceptible in comparison with the [distance to the stars].

5. Whatever motion appears in the [sky] arises not from any motion of the [sky], but from the earth's motion. The earth together with its [water and air] performs a complete rotation on its fixed poles in a daily motion, while the [sky remains] unchanged.

6. What appear to us as motions of the sun arise not from its motion but from the motion of the earth and...we revolve about the sun like any other planet. The earth has, then, more than one motion.

7. The apparent retrograde...motion of the planets arises not from their motion but from the earth's. The motions of the earth alone, therefore, are sufficient to explain so many apparent [motions] in the [sky]."

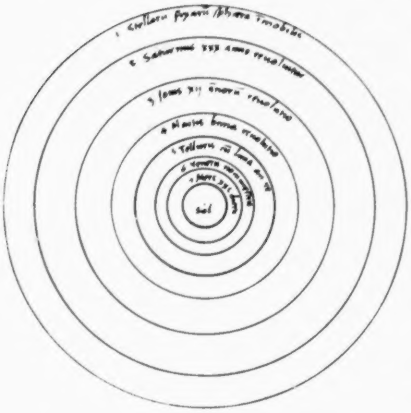

1 Stellarum fixarum sphaera immobilis
2 Saturnus XXX anno revolutionis
3 Jovis XII annorum revolutio
4 Martis bima revolutio
sol

Fig. 6.2 Copernicus' diagram of his heliocentric system. (From his manuscript of De Revolutionibus, 1543.) This simplified representation omits the many epicycles actually used in the system.

Comparison of this list with the assumptions of Ptolemy, given in Chapter 5, will show some identities as well as some important differences.

Notice that Copernicus proposed that the earth rotates daily. As Aristarchus and others had realized, this rotation would account for all the daily risings and settings observed in the sky. Also Copernicus proposed, as Aristarchus had done, that the sun was stationary and the center of the universe. The earth, like the other planets, moved about some point near the sun. Thus the Copernican system is a heliostatic (fixed sun) system in which the sun is located near, but not at, the various centers around which the planets moved.

Figure 6.2 shows the main concentric spheres carrying the planets around the sun. His text explains the outlines of the system:

> The ideas here stated are difficult, even almost impossible, to accept; they are quite contrary to popular notions. Yet with the help of God, we will make everything as clear as day in what follows, at least for those who are not ignorant of mathematics....
>
> The first and highest of all the spheres is the sphere of the fixed stars. It encloses all the other spheres and is itself self-contained; it is immobile; it is certainly the portion of the universe with reference to which the movement and positions of all the other heavenly bodies must be considered. If some people are yet of the opinion that this sphere moves, we are of a contrary mind; and after deducing the motion of the earth, we shall show why we so conclude. Saturn, first of the planets, which accomplishes its revolution in thirty years, is nearest to the first sphere. Jupiter, making its revolution in twelve years, is next. Then comes Mars, revolving once in two years. The fourth place in the series is occupied by the sphere which contains the earth and the sphere of the moon, and which performs an annual revolution. The fifth place is that of Venus, revolving in nine months. Finally, the sixth place is occupied by Mercury, revolving in eighty days.
>
> In the midst of all, the sun reposes, unmoving. Who, indeed, in this most beautiful temple would place the light-giver in any other part than that whence it can illumine all other parts...?
>
> In this ordering there appears a wonderful symmetry in the world and a precise relation between the motions and sizes of the spheres which no other arrangement offers.

Q1 What reason did Copernicus give for rejecting the use of equants?

Q2 What could Copernicus have meant when he said, "There is no one center of all the celestial circles or spheres," and yet "All the spheres revolve around the sun as their mid-point, and therefore the sun is the center of the universe"?

Q3 In the following table mark with a P the assumptions made by Ptolemy, and with a C those made by Copernicus.

a) The earth is spherical.

b) The earth is only a point compared to the distances to the stars.

c) The heavens rotate daily around the earth.

d) The earth has one or more motions.

e) Heavenly motions are circular.

f) The observed retrograde motion of the planets results from the earth's motion around the sun.

6.2 New conclusions. As often happens in science, a new way of looking at the observations—a new theory—leads to new types of conclusions. Copernicus used his moving-earth model to get two results not possible with the Ptolemaic theory. He found the periods of motion of each planet around the sun. Also he found the distance of each planet from the sun in terms of the distance of the earth from the sun. The distance between the earth and sun is known as the astronomical unit (A.U.).

If a body is observed to make 6 full cycles of the sky in 24 years, what approximately is its orbital period?

To get the periods of the planets around the sun Copernicus used observations that had been recorded over many years. For the outer planets, Mars, Jupiter and Saturn, he found the average number of years needed for the planet to make one trip around the sky, as Table 6.1 shows. When averaged over many years the period is rather close to the planet's actual orbital period.

Table 6.1 Copernicus' Derivation of the Period of Mars, Jupiter and Saturn around the Sun.

Planet	Years of Obs.	Cycles among the Stars	Ratio	Period Copernicus'	Period Modern
Mars	79	42	79/42	687d	687.0d
Jupiter	71	6	71/6	11.8y	11.86y
Saturn	59	2	59/2	29.5y	29.46y

For the quick-moving inner planets, Mercury and Venus, the procedure of Copernicus had a form which we call the "chase problem." As an example of such a chase, consider the hour and minute hands of a clock or watch, as shown in

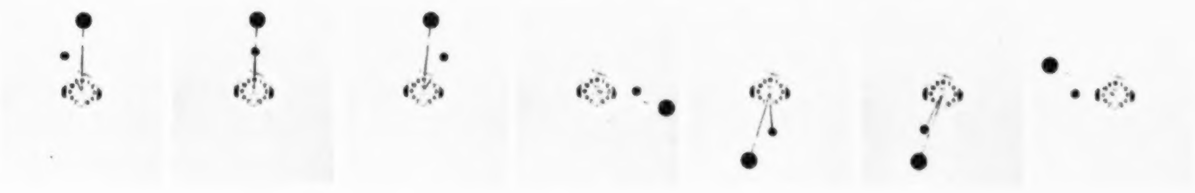


Fig. 6.3 Clock analogy of the "chase" problem. The small disk, representing the earth, is on an extension of the minute hand. The larger disk, representing a planet, is on an extension of the hour hand. The sun is at the center. The sequence shows the earth overtaking and passing the planet.

(a) 11:55
(b) 12:00
(c) 12:05
(d) 3:15
(e) 6:30
(f) 6:35
(g) 9:45

Fig. 6.3. If you were riding on the long hour hand shown in Fig. 6.3, how many times in 12 hours would the minute hand pass between you and the center? If you are not certain, slowly turn the hands of a clock or watch and keep count. From this information, can you derive a relation by which you would conclude that the period of the minute hand around the center was one hour?

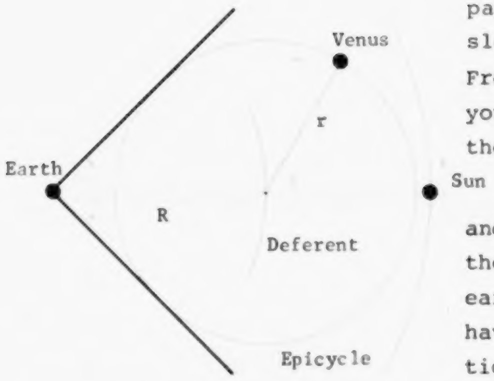


Fig. 6.4(a) The orbit of Venus according to Ptolemy. The maximum angle between the sun and Venus is about 45° east and west of the line connecting the fixed earth and the revolving sun. r = 0.7R

Now for a planetary application. We assume that Mercury and Venus are closer than the earth is to the sun, and that they have orbital periods less than one year. Because the earth is moving in the same direction as the planets, they have to chase the earth to return to the same apparent position in the sky—such as being farthest eastward from the sun. We can solve such a chase problem by counting for an interval of T years the number of times N a planet attains some particular position relative to the sun. The actual number of trips the planet has made around the sun in this interval of T years is the sum of N and T. The planet's period, years per revolution, is then T/(T + N). From observations available to him Copernicus formed the ratios T/(T + N) and found the periods shown in Table 6.2. His results were remarkably close to our present values.

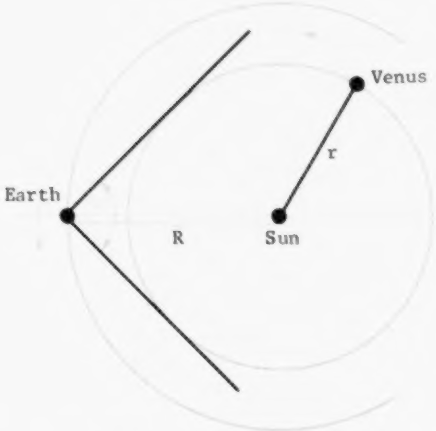


Fig. 6.4(b) The orbit of Venus, according to Copernicus. With the earth in orbit around the sun the same maximum angle between Venus and the earth-sun line is observed. Here r = 0.7R in astronomical units.

Table 6.2 Copernicus' Derivation of Periods of Mercury and Venus around the Sun

Planet	Years of Orbits T	Number of Times Farthest East of the Sun N	(T+N)	T/(T+N)	Period Copernicus'	Period Modern
Mercury	46	145	191	46/191y	88d	88.0d
Venus	8	5	13	8/13y	224d	224.7d

For the first time in history, Copernicus was able to derive distances to the planets in terms of the distance of the earth from the sun (the astronomical unit). Remember that the Ptolemaic system had no distance scale; it provided only a way of deriving the directions to the planets. If, as Copernicus proposed, the earth moved around the sun, the distances of the inner planets to the sun could be found in terms of the earth's distance, as Fig. 6.4 indicates, from their maximum angles from the sun. The values found by Copernicus (with the modern values in parentheses) are: Venus 0.72 (0.72 A.U.), Mercury 0.38 (0.39 A.U.). The earth's distance from the sun has been taken as 1.00, or one A.U.

In the Copernican system, the large epicycles of Ptolemy, shown in Fig. 5.15, p. 22, were replaced by the orbit of the earth. The radius r of Ptolemy's epicycle was given by Copernicus in terms of the radius R of the deferent, which was taken as 10,000 (see Table 6.3). From these numbers we can find for each planet the ratio of the radius of the deferent to that of the epicycle; these are listed in the third column of Table 6.3.

As Figs. 6.5(a) and (b) show, we can in imagination expand the scale of the planet's orbit (according to Ptolemy), both the deferent and epicycle, until the radius r of the epicycle is the same size as the orbit of the sun about the earth (or the earth about the sun). Figure 6.5c shows that we can then displace all three bodies: planet, sun and earth, along parallel lines and through equal distances. By this displace-

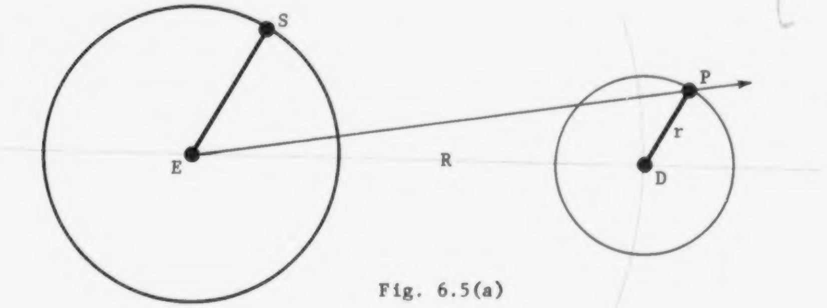


Fig. 6.5(a)

Fig. 6.5(a) The orbit of the sun S around the earth E and the deferent and epicycle of an outer planet P, as shown in Fig. 5.15.

(b) The deferent and epicycle are enlarged while maintaining the same maximum angle of displacement for the epicycle until the epicycle has the same radius, r', as ES, the earth-to-sun distance.

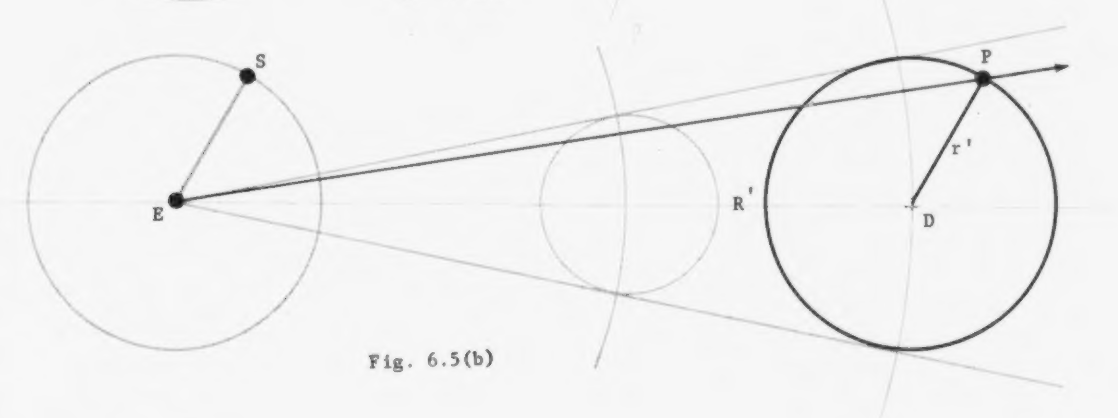


Fig. 6.5(b)

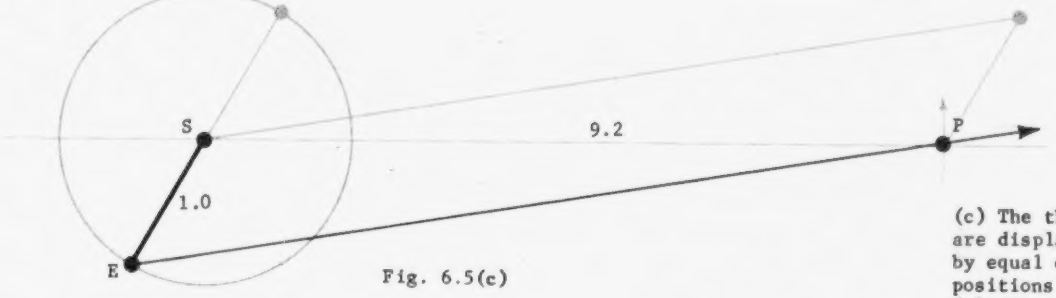


Fig. 6.5(c)

(c) The three bodies E, S and P are displaced on parallel lines by equal distances. The relative positions are the same as in part (a), but now the sun is the center of the system and the earth's orbit replaces the epicycle of the planet P.

ment we move the earth from the center of all the motions and put the sun at the center. Also, we have eliminated the planet's epicycle and replaced it by the orbit of the earth. In addition, we have changed the frame of reference and put the sun instead of the earth at the origin of the coordinate system and the center of the planetary motions. Now the model resembles the Copernican system. Furthermore, the relative distance from the sun to each planet is set.

Table 6.3 The Sizes of Planetary Orbits

	Ptolemy's Ratios Deferent/Epicycle (R/r)	Copernicus' R, when r = 1	Modern Values
Saturn	10,000/1090	9.2	9.54 A.U.
Jupiter	10,000/1916	5.2	5.20
Mars	10,000/6580	1.52	1.52
Earth	(one astronomical unit)	1.00	1.00
Venus		0.72	0.72
Mercury		0.38	0.39

Q4 What new results did Copernicus obtain with a moving-earth model which were not possible with a geocentric model for the planetary system?

6.3 Arguments for the Copernican System.

Since Copernicus knew that to many his work would seem absurd, "nay, almost contrary to ordinary human understanding," he tried to meet the old arguments against a moving earth in several ways.

1. Copernicus argued that his assumptions agreed with dogma at least as well as Ptolemy's. Copernicus has many sections on the limitations of the Ptolemaic system (most of which had been known for centuries). Other sections pointed out how harmonious and orderly his own system seems and how pleasingly his system reflects the mind of the Divine Architect. To Copernicus, as to many scientists, the complex events they saw were but symbols of the working of God's mind. To seek symmetry and order in the observed changes was to Copernicus an act of reverence. To him the symmetry and order were renewed proof of the existence of the Deity. As a highly placed and honored church dignitary, he would have been horrified if he had been able to foresee that his theory would contribute to the sharp clash, in Galileo's time, between religious dogma and the interpretations that scientists gave to their experiments.

2. Copernicus' analysis was as thorough as that of Ptolemy. He calculated the relative radii and speeds of the circular motions in his system so that tables of planetary motion could be made. Actually the theories of

Ptolemy and Copernicus were about equally accurate in predicting planetary positions, which for both theories often differed from the observed positions by as much as 2°, or four diameters of the moon.

3. Copernicus cleverly tried to answer several objections that were certain to be raised against his heliocentric system—as they had been, long ago, against that of Aristarchus. To the argument that the earth, rotating so rapidly about its own axis, would surely burst like a flywheel driven too fast, he asked, "Why does the defender of the geocentric theory not fear the same fate for his rotating celestial sphere—so much faster because so much larger?" To the argument that birds in flight and clouds should be left behind by the rapidly rotating and revolving earth, he answered that the atmosphere is dragged along with the earth.

To the old question of the absence of parallax for the fixed stars, he could only give the same answer as Aristarchus:

> ...the dimensions of the world [universe] are so vast that though the distance from the sun to the earth appears very large as compared with the size of the spheres of some planets, yet compared with the dimensions of the sphere of the fixed stars, it is as nothing.

However, would you expect that those who believed in a small earth-centered universe would be persuaded that the stars were far away because their parallax was not observed? The argument was logical, but not convincing.

4. Copernicus claimed that the greatest advantage of his scheme was its simple description of the general motions of the planets. Figure 5.7, p. 11, shows how the retrograde motions will appear from a moving earth.

Yet for computations, because Copernicus would not use equants, he needed more small motions than did Ptolemy to account for the observations.

Q5 What arguments did Copernicus use in favor of his system?

Q6 What were the largest differences between observed planetary positions and those predicted by Ptolemy and Copernicus?

Q7 In what way was Copernicus' conclusion about the distance to the stars not convincing?

Q8 Did the Copernican system provide simple calculations of where the planets should be seen?

6.4 Arguments against the Copernican system. Copernicus' hopes for acceptance of his theory were not quickly fulfilled. More than a hundred years passed before the heliocentric system was generally accepted even by astronomers. In the meantime the theory and its few champions met powerful opposition. Most of the arguments were the same as those used by Ptolemy against the heliocentric ideas of Aristarchus.

1. Apart from its apparent simplicity, the Copernican system had no scientific advantages over the geocentric theory. There was no known observation that was explained by one system and not by the other. Copernicus introduced no new types of observations into his work. Furthermore, the accuracy of his final predictions was little better than that of Ptolemy's results. As Francis Bacon wrote in the early seventeenth century: "Now it is easy to see that both they who think the earth revolves and they who hold the old construction are about equally and indifferently supported by the phenomena."

Basically, the rival systems differed in their choice of reference systems used to describe the observed motions. Copernicus himself stated the problem clearly:

> Although there are so many authorities for saying that the Earth rests in the centre of the world that people think the contrary supposition...ridiculous; ...if, however, we consider the thing attentively, we will see that the question has not yet been decided and accordingly is by no means to be scorned. For every apparent change in place occurs on account of the movement either of the thing seen or of the spectator, or on account of the necessarily unequal movement of both. For no movement is perceptible relatively to things moved equally in the same directions—I mean relatively to the thing seen and the spectator. Now it is from the Earth that the celestial circuit is beheld and presented to our sight. Therefore, if some movement should belong to the Earth...it will appear, in the parts of the universe which are outside, as the same movement but in the opposite direction, as though the things outside were passing over. And the daily revolution...is such a movement.

In that statement Copernicus invites the reader to shift the frame of reference from that of an observer on the earth to one at a remote position looking upon the whole system with the sun at the center. As you may know from personal experience, such a shift is not easy for us even today. Perhaps you can sympathize with those who preferred to hold to an earth-centered system for describing what they actually saw.

Physicists now generally agree that all systems of reference are equivalent, although some may be more complex to use or think about. The modern attitude is that the choice of a frame of reference depends mainly on which will provide the simplest solution to the problem being studied. We should not speak of reference systems as being right or wrong, but rather as being convenient or inconvenient. However, a reference system that may be acceptable to one person may involve philosophical assumptions that are unacceptable to another.

2. The lack of an observable parallax for the fixed stars was against Copernicus' model. His only reply was unacceptable because it meant that the stars were practically an infinite distance away from the earth. To us this is no shock, because we have been raised in a society that accepts the idea. Even so, such distances do strain our imagination. To the opponents of Copernicus such distances were absurd.

The Copernican system led to other conclusions that were also puzzling and threatening. Copernicus found actual distances between the sun and the planetary orbits. Perhaps the Copernican system was not just a mathematical procedure for predicting the positions of the planets. Perhaps Copernicus had revealed a real system of planetary orbits in space. This would be most confusing, for the orbits were far apart. Even the few small epicycles needed to account for variations in the motions did not fill up the spaces between the planets. Then what did fill up these spaces? Because Aristotle had stated that "nature abhors a vacuum," there had to be something in all that space. As you might expect, those who felt that space should be full of something invented various sorts of invisible fluids and ethers to fill up the emptiness. More recently analogous fluids have been used in theories of chemistry, and of heat, light and electricity.

3. Since no definite decision between the Ptolemaic and the Copernican theories could be made on the astronomical evidence, attention focused on the argument concerning the immobility and central position of the earth. For all his efforts, Copernicus was unable to persuade most of his readers that the heliocentric system was at least as close as the geocentric system to the mind and intent of God. All religious faiths in Europe, including the new Protestants, found enough Biblical quotations (e.g., Joshua 10: 12-13) to assert that the Divine Architect had worked from a Ptolemaic blueprint. Indeed, the religious reformer Martin Luther branded Copernicus as "the fool who would overturn the whole science of astronomy."

Eventually, in 1616, when storm clouds were raised by the case of Galileo, the Papacy put De Revolutionibus on the Index of forbidden books as "false and altogether opposed to Holy Scriptures," and withdrew its approval of an earlier outline of Copernicus' work. Some Jewish communities forbade the teaching of the heliocentric theory. It was as if man insisted on the middle of the stage for his earth, the scene of both his daily life and prayer in a world he felt was created especially for him.

The assumption that the earth was not the center of the universe was offensive. But worse, the Copernican system suggested that the other planets were similar to the earth. Thus the concept of the heavenly ether was threatened. Who knew but what some fool might next suggest that the sun and possibly even the stars were made of earthly materials? If the other celestial bodies, either in our solar system or beyond, were similar to the earth, they might even be inhabited, no doubt by heathens beyond the power of salvation! Thus the whole Copernican scheme led to profound philosophical questions.

4. The Copernican theory conflicted with the basic propositions of Aristotelian physics. This conflict is well described by H. Butterfield:

> ...at least some of the economy of the Copernican system is rather an optical illusion of more recent centuries. We nowadays may say that it requires smaller effort to move the earth round upon its axis than to swing the whole universe in a twenty-four hour revolution about the earth; but in the Aristotelian physics it required something colossal to shift the heavy and sluggish earth, while all the skies were made of a subtle substance that was supposed to have no weight, and they were comparatively easy to turn, since turning was concordant with their nature. Above all, if you grant Copernicus a certain advantage in respect of geometrical simplicity, the sacrifice that had to be made for the sake of this was tremendous. You lost the whole cosmology associated with Aristotelianism—the whole intricately dovetailed system in which the nobility of the various elements and the hierarchical arrangement of these had been so beautifully interlocked. In fact, you had to throw overboard the very framework of existing science, and it was here that Copernicus clearly failed to discover a satisfactory alternative. He provided a neater geometry of the heavens, but it was one which made nonsense of the reasons and explanations that had previously been given to account for the movements in the sky.

Although the sun-centered Copernican scheme was equivalent to the Ptolemaic in explaining the astronomical observations, to abandon the geocentric hypothesis seemed "philosophically false and absurd," dangerous, and fantastic. What other reaction could one have expected? Learned Europeans at that time recognized the Bible and the writings of Aristotle as their two supreme sources of authority. Both appeared to be challenged by the Copernican system. Although the freedom of thought that marked the Renaissance was just beginning, the old image of the world provided security and stability to many.

Similar conflicts between the philosophical assumptions underlying accepted beliefs and those arising from scientific studies have occurred many times. During the last century there were at least two such conflicts. Neither is completely resolved today. In biology the evolutionary theory based on Darwin's work has had major philosophical and religious overtones. In physics, as Units 4, 5 and 6 indicate, evolving theories of atoms, relativity, and quantum mechanics have challenged other long-held philosophical assumptions about the nature of the world and our knowledge of reality.

Q9 Why did many people, such as Francis Bacon, adopt a ho-hum attitude toward the arguments about the correctness of the Ptolemaic or Copernican systems?

Q10 What was the major difference between the Ptolemaic and the Copernican systems?

Q11 How did the astronomical argument become involved with religious beliefs?

Q12 In what way did the Copernican system conflict with the accepted physics of the time?

Q13 List some conflicts between scientific theories and philosophical assumptions of which you are aware.

6.5 Historical consequences. Eventually, the moving-earth model of Copernicus was accepted. However, the slowness of that acceptance is illustrated by a recent discovery in the diary of John Adams, the second President of the United States: he wrote that at Harvard College on June 19, 1753, he attended a lecture where the correctness of the Copernican system was disputed.

Soon we shall follow the work which gradually led to the general acceptance of the heliocentric theory. Yet within a century the detailed Copernican system of uniform circular motions with eccentrics and epicycles was replaced. We shall see that the real scientific significance of Copernicus' work lies in the fact that a heliocentric formulation opened a new way for understanding planetary motion. This was through the simple laws of ordinary (terrestrial) mechanics which were developed during the 150 years that followed.

The Copernican model with moving earth and fixed sun opened a floodgate of new possibilities for analysis and description. According to this model the planets could be considered as real bodies moving along actual orbits.

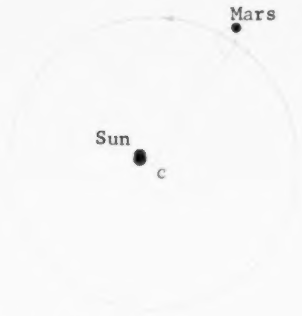


Fig. 6.6 In the Copernican system, the center of the deferent of Mars was offset from the sun to point C. In addition, a small epicycle was needed to account for minor variations in the planet's motion.

Now Kepler and others could consider these planetary paths in quite new ways.

In science, a new set of assumptions often leads to new interpretations and unexpected results. Usually the sweep of possibilities cannot be foreseen by those who begin the revolution—or by their critics. For example, the people who laughed at the first automobiles, which moved no faster than a walking horse, failed to realize that those automobiles were but a crude beginning and could soon be improved, while the horse was in its "final edition."

The memory of Copernicus is honored for two additional reasons. First, he was one of those giants of the fifteenth and sixteenth centuries who challenged the contemporary world-picture. Second, his theory became a main force in the intellectual revolution which shook man out of his self-centered view of the universe.

As men gradually accepted the Copernican system, they necessarily found themselves accepting the view that the earth was only one among several planets circling the sun. Thus it became increasingly difficult to assume that all creation centered on mankind.

What other scientific theories do you know which challenge the assumption that man is the summit of creation?

Acceptance of a revolutionary idea based on quite new assumptions, such as Copernicus' shift of the frame of reference, is always slow. Sometimes compromise theories are proposed as attempts to unite two conflicting alternatives, that is, "to split the difference." As we shall see in various Units, such compromises are rarely successful. Yet the conflict usually stimulates new observations that may be of long-term importance. These may lead to the development or restatement of one theory until it is essentially a new theory, as we shall see in Chapter 7.

Such a restatement of the heliocentric theory came during the years after Copernicus. While many men provided observations and ideas, we shall see that major contributions were made by Tycho Brahe, Kepler, Galileo and then Isaac Newton. New and better solutions to the theoretical problems required major improvements in the precision with which planetary positions were observed. Such improvements and the proposal of a compromise theory were the life work of the astronomer Tycho Brahe.

Q14 In terms of our historical perspective, what was probably the greatest contribution of Copernicus?

Q15 How did the Copernican system encourage the suspicion that there might be life on objects other than the earth? Is such a possibility seriously considered today?

6.6 Tycho Brahe. Tycho Brahe (Fig. 6.7) was born in 1546 of a noble, but not particularly rich, Danish family. By the time Tycho was thirteen or fourteen, he had become intensely interested in astronomy. Although he was studying law, he secretly spent his allowance money on astronomical tables and books such as the Almagest and De Revolutionibus. Soon he discovered that both Ptolemy and Copernicus had relied upon tables of planetary positions that were inaccurate. He concluded that before a satisfactory theory of planetary motion could be created new astronomical observations of the highest possible accuracy gathered during many years would be necessary.

See "The Boy Who Redeemed His Father's Name" in Project Physics Reader 2.

Tycho's interest in studying the heavens was increased by an exciting observation in 1572. Although the ancients had taught that the stars were unchanging, Tycho observed a "new star" in the constellation Cassiopeia. It soon became as bright as Venus and could be seen even during the daytime. Then over several years it faded until it was no longer visible. To Tycho these changes were astonishing: change in the starry sky! Since the ancients had firmly believed that no changes were possible in the starry heavens, at least one assumption of the ancients was wrong. Perhaps other assumptions were wrong, too. What an exciting life he might have if he could study the heavens, searching for other changes of the stars and planets.

See "The Great Comet of 1965" in Project Physics Reader 2.

After observing and writing about the "new star," Tycho travelled through northern Europe where he met many other astronomers and collected books. Apparently he was considering moving to Germany or Switzerland where he could easily meet other astronomers. To keep the young scientist in Denmark, King Frederick II made Tycho an offer that was too attractive to turn down. Tycho was given an entire small island, and also the income derived from various farms to allow him to build an observatory on the island and to staff and maintain it. The offer was accepted, and in a few years Uraniborg ("Castle of the Heavens") was built (Fig. 6.8). It was a large structure, having four large observatories, a library, a laboratory, shops, and living quarters for staff, students and observers. There was even a complete printing plant. Tycho estimated that the observatory cost Frederick II more than a ton of gold. In terms of the time of its building, this magnificent laboratory was at least as significant, complex and expensive as some of the great research establishments of our own time. Primarily

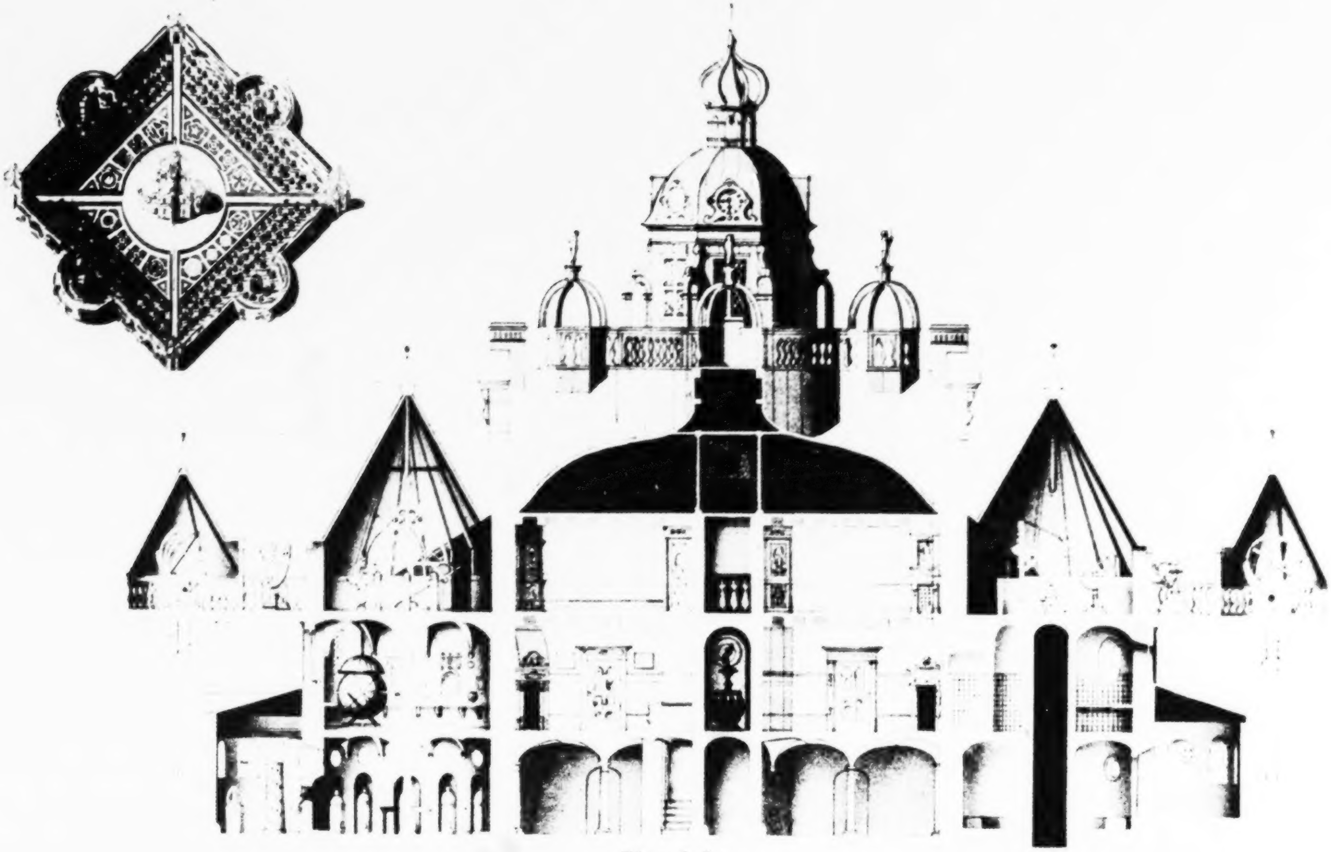
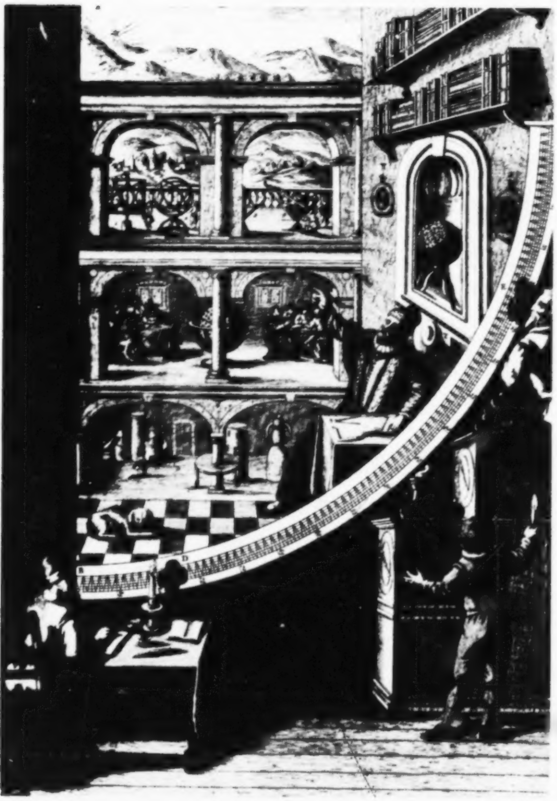


Fig. 6.8



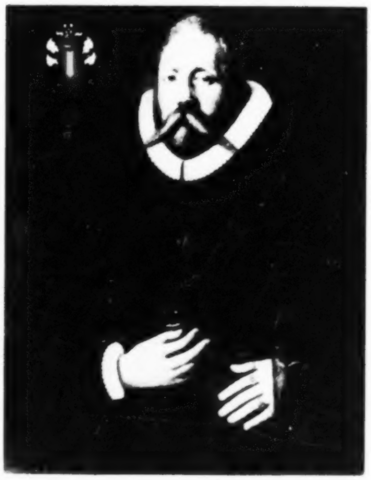


Fig. 6.7

At the top left is a plan of the observatory, gardens and wall built for Tycho Brahe at Uraniborg, Denmark.

The cross section of the observatory, above center, shows where most of the important instruments were housed. Under the arch near the left is Tycho's largest celestial sphere.

At the left is the room containing Tycho's great quadrant. On the walls are pictures of Tycho and some of his instruments.

Above is a portrait of Tycho painted about 1597.

a research center, Uraniborg was a place where scientists, technicians and students from many lands could gather to study astronomy. Here was a unity of action, a group effort under the leadership of an imaginative scientist to advance the boundaries of knowledge in one science.

In 1577 Tycho observed a bright comet, a fuzzy object which moved across the sky erratically, unlike the orderly motions of the planets. To find the distance to the comet Tycho compared observations of its position seen from Denmark with its positions observed from elsewhere in Europe. At a given time, the comet had the same position among the stars even though the observing places were many hundreds of miles apart. Yet the moon's position in the sky was different when viewed from the ends of such long baselines. Therefore, Tycho concluded, the comet must be at least six times farther away than the moon. This was an important conclusion. Up to that time comets had been believed to be some sort of local event, like a cloud in the sky. Now comets had to be considered as distant astronomical objects which seemed to move right through the crystalline spheres. Tycho's book on this comet was widely read and helped to undermine belief in the old assumptions about the nature of the heavens.

Fig. 6.9 The bright comet of 1965.

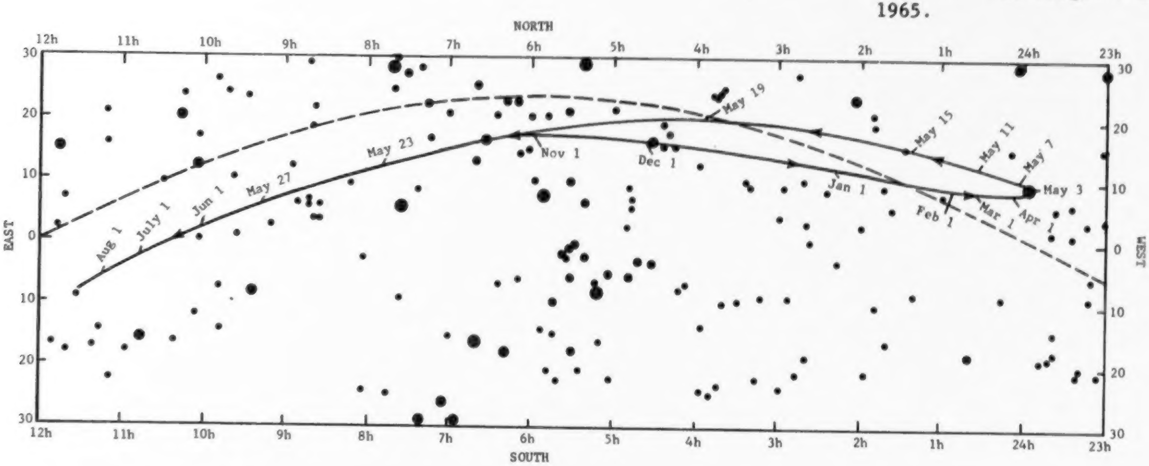


Fig. 6.10 **Motion of Halley's Comet in 1909-10.**

Q16 What stimulated Tycho to become interested in astronomy?

Q17 Why were Tycho's conclusions about the comet of 1577 important?

Q18 In what ways was Tycho's observatory like a modern research institute?

Q19 What evidence can you find that comets had been considered as omens of some disaster?

Q20 How can you explain the observed motion of Halley's comet during 1909-1910 as shown in Fig. 6.10?

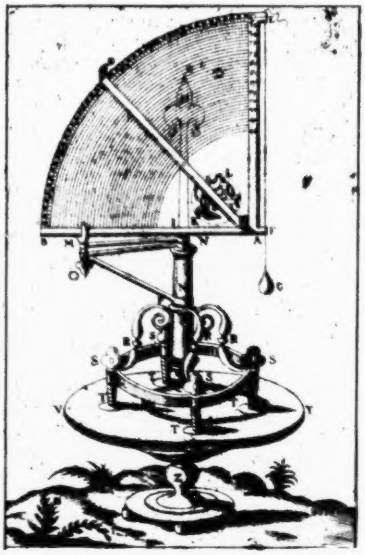


Fig. 6.11 One of Tycho's instruments, a quadrant; a device for measuring the angular altitude of heavenly objects. Unfortunately all of Tycho's instruments have been destroyed or lost.

See "A Night at the Observatory" in Project Physics Reader 2.

For a more modern example of this same problem of instrumentation, you may wish to read about the development and construction of the 200-inch Hale telescope on Mt. Palomar.

6.7 Tycho's observations. Tycho's fame results from his lifelong devotion to making unusually accurate observations of the positions of the stars, sun, moon, and planets. He did this before the telescope was invented. Over the centuries many talented observers had been recording the positions of the celestial objects, but the accuracy of Tycho's work was much greater than that of the best astronomers before him. How was Tycho Brahe able to do what no others had done before?

Singleness of purpose was certainly one of Tycho's assets. He knew that observations of the highest precision must be made during many years. For this he needed improved instruments that would give consistent readings. Fortunately he possessed both the mechanical ingenuity to devise such instruments and the funds to pay for their construction and use.

Tycho's first improvement on the astronomical instruments of the day was to make them larger. Most of the earlier instruments had been rather small, of a size that could be moved by one person. In comparison, Tycho's instruments were gigantic. For instance, one of his early devices for measuring the angular altitude of planets was a quadrant having a radius of about six feet (Fig. 6.11). This wooden instrument was so large that it took many men to set it into position. Tycho also put his instruments on heavy, firm foundations. Another huge instrument was attached to a wall that ran exactly north-south. By increasing the stability of the instruments, Tycho increased the reliability of the readings over long periods of time. Throughout his career Tycho also created better sighting devices, more precise scales and stronger support systems, and made dozens of other changes in design which increased the precision of the observations.

Not only did Tycho devise better instruments for making his observations, but he also determined and specified the actual limits of precision of each instrument. He realized that merely making larger and larger instruments does not always result in greater precision; ultimately, the very size of the instrument introduces errors since the parts will bend under their own weight. Tycho therefore tried to make his instruments as large and strong as he could without at the same time introducing errors due to bending. Furthermore, in the modern tradition, Tycho calibrated each instrument and determined its range of systematic error. (Nowadays most scientific instruments designed

for precision work are accompanied by a statement, usually in the form of a table, of systematic corrections to be applied to the readings.)

Like Ptolemy and the Moslem observers, Tycho knew that the light from each heavenly body was bent downward increasingly as the object neared the horizon (Figs. 6.12 and 6.13), an effect known as atmospheric refraction. To increase the precision of his observations, Tycho carefully determined the amount of refraction so that each observation could be corrected for refraction effects. Such careful work was essential if improved records were to be made.

At Uraniborg, Tycho worked from 1576 to 1597. After the death of King Frederick II, the Danish government became less interested in helping to pay the cost of Tycho's observatory. Yet Tycho was unwilling to consider any reduction in the costs of his activities. Because he was promised support by Emperor Rudolph of Bohemia, Tycho moved his records and several instruments to Prague. There, fortunately, he took on as an assistant an able, imaginative young man named Johannes Kepler. When Tycho died in 1601, Kepler obtained all his records about Mars. As Chapter 7 reports, Kepler's analysis of Tycho's observations solved much of the ancient problem.

Fig. 6.12 The oblate setting sun. The light's path through the earth's atmosphere caused the sun to appear both oval and rough-edged.

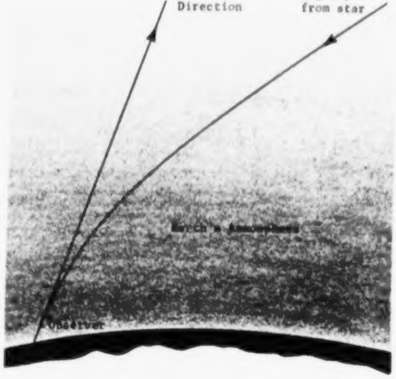


Fig. 6.13 Refraction, or bending, of light from a star by the earth's atmosphere. The amount of refraction shown in the figure is greatly exaggerated over what actually occurs.

Q21 What improvements did Tycho make in astronomical instruments?

Q22 In what way did Tycho correct his observations to provide records of higher accuracy?

6.8 Tycho's compromise system. Tycho's observations were intended to provide a basis for a new theory of planetary motion which he had outlined (Fig. 6.14) in an early publication. Tycho saw the simplicity of the Copernican system by which the planets moved around the sun, but he could not accept the idea that the earth had any motion. In Tycho's system, all the planets except the earth moved around the sun, which in turn moved around the stationary earth. Thus he devised a compromise theory which, as he said, included the best features of both the Ptolemaic and the Copernican systems, but he did not live to publish a quantitative theory. As we look at it today, his system is equivalent to either the Copernican or the Ptolemaic system. The difference between the three systems is the choice of what is regarded as stationary, that is, what frame of reference is chosen.

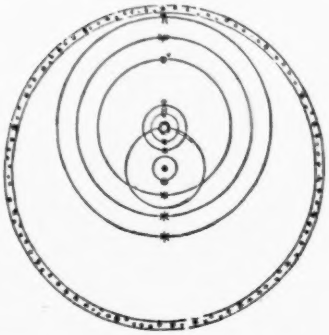


Fig. 6.14 Main spheres in Tycho Brahe's system of the universe. The earth was fixed and was at the center of the universe. The planets revolved around the sun, while the sun, in turn, revolved around the fixed earth.

The compromise Tychonic system was accepted by some people and rejected by others. Those who accepted Ptolemy objected to Tycho's proposal that the planets moved around the sun. Those who were interested in the Copernican model objected to having the earth held stationary. Thus the argument continued between those holding the seemingly self-evident position that the earth was stationary and those who accepted, at least tentatively, the strange, exciting proposals of Copernicus that the earth might rotate and revolve around the sun. These were philosophical or aesthetic preferences, for the scientific evidence did not yet provide an observational basis for a choice. To resolve the conflict and to produce a drastically revised sun-centered model was the work of Kepler who analyzed Tycho's high-quality observations of Mars.

All planetary theories up to this time had been developed only to provide some system by which the positions of the planets could be predicted fairly precisely. In the terms used in Unit I, these would be called kinematic descriptions. The causes of the motions—what we now call the dynamics of the motions—had not been questioned. The motions were, as Aristotle said, "natural." The heavens were still considered to be completely different from earthly materials and to change in quite different ways. That a common physics could describe both earthly and heavenly motions was a revolutionary idea yet to be proposed.

The Copernican system opened again the argument mentioned at the end of Chapter 5: were the Copernican orbits actual paths in space, or only convenient computational devices? We shall see that the eventual success of the Newtonian synthesis led to the confident assumption that scientists were describing the real world. However, later chapters of this text, dealing with recent discoveries and theories, will indicate that today scientists are much less certain that they know what is meant by the word reality.

The status of the problem in the early part of the seventeenth century was later well described by the English poet, John Milton, in Paradise Lost:

> ...He his fabric of the Heavens
> Hath left to their disputes, perhaps to move
> His laughter at their quaint opinions wide
> Hereafter, when they come to model Heaven
> And calculate the stars, how they will wield
> The mighty frame, how build, unbuild, contrive
> To save appearances, how gird the sphere
> With centric and eccentric scribbled o'er,
> Cycle and epicycle, orb in orb.

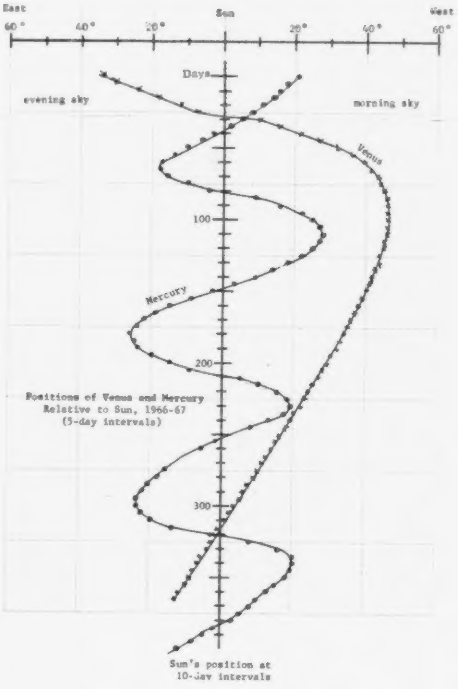
Q23 In what ways did Tycho's system for planetary motions resemble either the Ptolemaic or the Copernican systems?

Q24 To what degree do you feel that the Copernican system, with its many motions on eccentrics and epicycles, reveals real paths in space rather than being only another means of computing planetary positions?

Study Guide

6.1 The diagram to the right shows the motions of Mercury and Venus east and west of the sun as seen from the earth during 1966-67. The time scale is indicated at 10-day intervals along the central line of the sun's position.

a) Can you explain why Mercury and Venus appear to move from farthest east to farthest west more quickly than from farthest west to farthest east?
b) From this diagram can you find a period for Mercury's apparent position in the sky relative to the sun?
c) With the aid of the "watch model" can you derive a period for Mercury's orbital motion around the sun?
d) What are the major sources of uncertainty in the results you derived?
e) Similarly can you estimate the orbital period of Venus?



Chapter 7 A New Universe Appears – The Work of Kepler and Galileo

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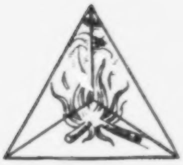

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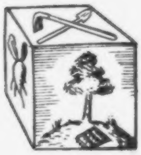

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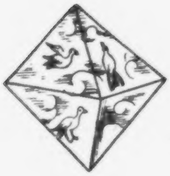

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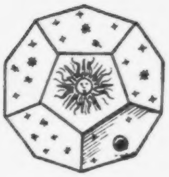

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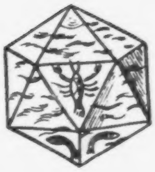
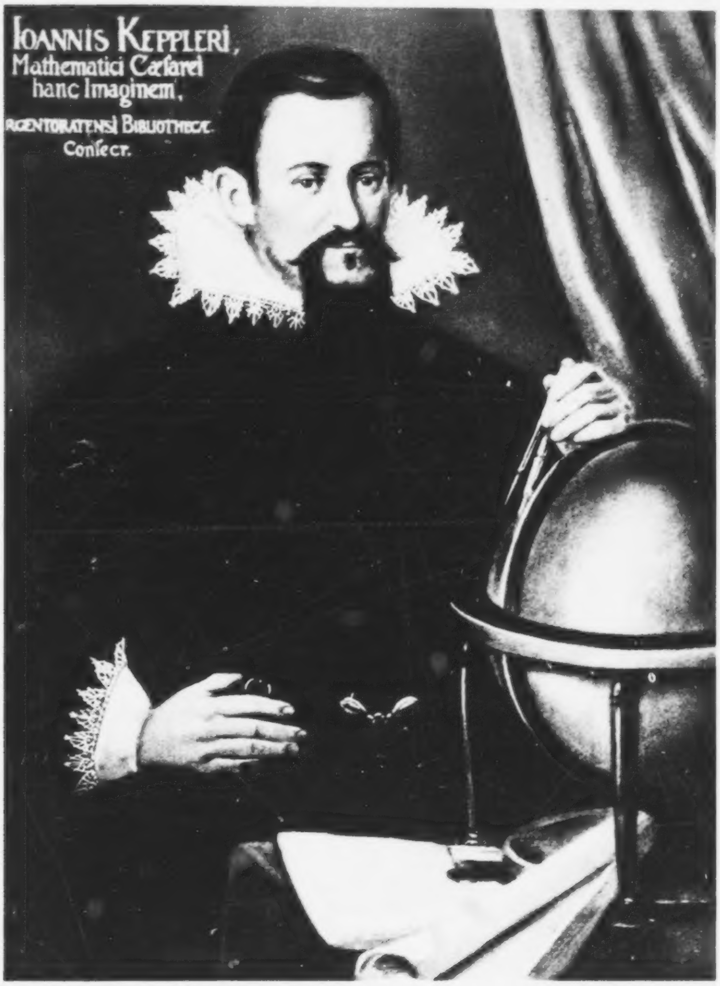



IOANNIS KEPPLERI,
Mathematici Cæſarei
hanc Imaginem,
RGENTORATENSI BIBLIOTHECÆ
Conſecr.

7.1 The abandonment of uniform circular motion. Kepler's lifelong desire was to perfect the heliocentric theory. He viewed the harmony and simplicity of that theory with "incredible and ravishing delight." To Kepler, Fig. 7.1, such patterns of geometric order and numerical relation were clues to God's mind. Therefore, to unfold these patterns further through the heliocentric theory Kepler attempted in his first major paper to explain the spacing of the planetary orbits, as found by Copernicus (Table 6.3, p. 34).

From the time of the Greeks learned men knew that there were just five regular geometrical solids. The regular solid with six square faces is the cube. The dodecahedron has twelve five-sided faces. The other three regular solids have faces which are equilateral triangles. The tetrahedron has four triangular faces, the octahedron has eight triangular faces, and the icosahedron has twenty triangular faces.

Kepler wondered whether there was any relation between these five solids and the six known planets. He realized that these five regular solids could be nested, one inside the other like a set of mixing bowls. Between the five solids would be four spaces for planetary spheres. A fifth sphere could be inside the whole nest and a sixth sphere could be around the outside. Perhaps then some sequence of the five solids just touching the spheres would actually be spaced at the same relative distances from the center as were the planetary orbits. Kepler said:

> I took the dimensions of the planetary orbits according to the astronomy of Copernicus, who makes the sun immobile in the center, and the earth movable both around the sun and upon its own axis; and I showed that the differences of their orbits corresponded to the five regular Pythagorean figures....

By trial and error Kepler found a fit within about five percent of the actual planetary distances. To Kepler this remarkable system, illustrated in Fig. 7.2, explained the spacings of the planets. Also it indicated the unity he expected between geometry and scientific observations. Kepler's results, published in 1597, demonstrated his imagination and computational ability. Furthermore, it brought him to the attention of Tycho and Galileo. As a result, Kepler was invited to become one of Tycho's assistants at his new observatory in Prague.

There Kepler was given the task of determining more precisely the orbit of Mars. This problem had not been solved by Tycho and his other assistants; the motion of Mars was unusually difficult to explain. Years later, after he had solved the problem, Kepler wrote that " . . . Mars alone

The five "perfect solids," taken from Kepler's Harmonices Mundi, are shown on the opposite page. You can distinguish several of these solids in Fig. 7.2. The five regular solids are

1. the tetrahedron: four equilateral triangles;
2. the cube: six squares;
3. the octahedron: eight equilateral triangles;
4. the dodecahedron: twelve pentagons;
5. the icosahedron: twenty equilateral triangles.

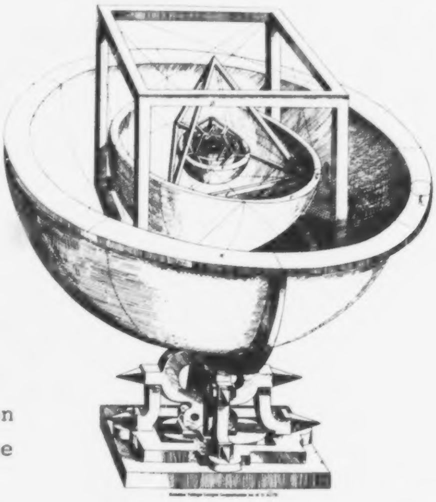


Fig. 7.2 A model of Kepler's explanation of the spacing of the planetary orbits by means of the regular geometrical solids. Notice that the planetary spheres were thick enough to include the small epicycle used by Copernicus.

Fig. 7.1 Johannes Kepler (1571-1630).

See "Kepler" in Project Physics Reader 2.

enables us to penetrate the secrets of astronomy which otherwise would remain forever hidden from us." The motion of Mars, it turned out, was the start from which Kepler could redirect the study of celestial astronomy, just as Galileo used the motion of falling bodies to redirect the study of terrestrial motion.

Kepler began his study of Mars by trying to fit the observations by means of motions on an eccentric circle and an equant. Like Copernicus, Kepler eliminated the need for the large epicycle by putting the sun motionless at the center and having the earth move around it, as in Fig. 6.5, p. 33. But Kepler made an assumption which differed from Copernicus' You recall that Copernicus had rejected the equant as an improper type of motion, but used small epicycles. Kepler used an equant, but refused to use even a small epicycle. To Kepler the epicycle seemed "unphysical" because the center of the epicycle was empty and empty space could not exert any force on a planet. Thus from the start of his study Kepler was assuming that the orbits were real and that the motions had some causes. Even though Kepler's teacher advised him to stick with "astronomical" (only observational) rather than physical assumptions, Kepler stubbornly stuck to his idea that the motions must have causes. When finally he published his results on Mars, in his book, Astronomia Nova, the New Astronomy, it was subtitled Celestial Physics.

For a year and a half Kepler struggled to fit Tycho's observations of Mars by various arrangements of an eccentric and an equant. When, after 70 trials, success finally seemed near he made a depressing discovery. Although he could represent fairly well the motion of Mars in longitude (along the ecliptic), he failed miserably with the latitude (the positions perpendicular to the ecliptic). However, even in longitude his very best fit still had differences of eight minutes of arc between the predicted and Tycho's observed positions.

See "Kepler on Mars" in Project Physics Reader 2.

Eight minutes of arc, about a fourth of the moon's diameter, may not seem like much of a difference. Others might have been tempted to explain it as observational error. But, with an integrity that has come to be expected of scientists, Kepler did not use that explanation. He knew from his own studies that Tycho's instruments and observations were rarely in error by as much as two minutes of arc. Those eight minutes of arc meant to Kepler that his best system using an eccentric and an equant would not do.

See "Kepler's Celestial Music" in Project Physics Reader 2.

In his New Astronomy Kepler wrote:

> Since divine kindness granted us Tycho Brahe, the most diligent observer, by whose observations an error of eight minutes in the case of Mars is brought to light in this Ptolemaic calculation, it is fitting that we recognize and honor this favor of God with gratitude of mind. Let us certainly work it out, so that we finally show the true form of the celestial motions (by supporting ourselves with these proofs of the fallacy of the suppositions assumed). I myself shall prepare this way for others in the following chapters according to my small abilities. For if I thought that the eight minutes of longitude were to be ignored, I would already have corrected the hypothesis found in Chapter 16 (that it, by bissecting the eccentricity). But as it is, because they could not be ignored, these eight minutes alone have prepared the way for reshaping the whole of astronomy, and they are the material which is made into a great part of this work.

The geometrical models of Ptolemy and Copernicus based on uniform circular motions had to be abandoned. Kepler had the finest observations ever made, but now he had no theory by which they could be explained. He would have to start over to account for the difficult questions: what is the shape of the orbit followed by Mars, and precisely how does the speed of the planet change as it moves along the orbit?

Q1 What brought Kepler to the attention of Tycho Brahe?

Q2 Why did Kepler conclude that Plato's problem, to describe the motions of the planets by combinations of circular motions, could not be solved?

7.2 Kepler's Law of Areas. Kepler's problem was immense. To solve it would demand the utmost of his imagination and computational skills, as well as of his persistence and health.

As the basis for his study Kepler had only Tycho's observed directions to Mars and to the sun on certain dates. But these observations were made from a moving earth whose orbit was not well known. Kepler realized that he must first determine more accurately the shape of the earth's orbit so that he would know where it was when the various observations of Mars had been made. Then he might be able to use the observations to determine the shape and size of the orbit of Mars. Finally, to predict positions for Mars he would need some regularity or law that described how fast Mars moved at various points in its orbit.

Fortunately Kepler made a major discovery which was crucial to his later work. He found that the orbits of the earth and other planets were in planes which passed through the sun. With this simplifying model of planets moving in individual planes Kepler could avoid the old patterns of Ptolemy and Copernicus which required separate explanations for the observed motions of the planets north and south of

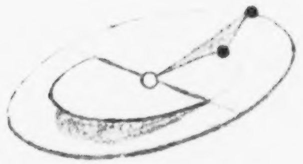


Fig. 7.3 Edge-on view of orbital planes of earth and planet.

the ecliptic. Kepler discovered that the changing positions of a planet could result from the planet's motion in its orbit viewed from the earth moving in its plane (the plane of the ecliptic) as shown in Fig. 7.3.

From his many studies Kepler knew that the earth and Mars moved in continuous paths that differed a bit from circles. His aim was to obtain a detailed picture or plot of the orbits as they might be seen by an observer above the ecliptic plane looking down on these moving bodies. To such an observer the planets would look like marbles rolling along nearly circular paths on the floor. Although the heliocentric idea gave a rough guide to what this system would look like, Kepler's task was to find from the data the general rules, or laws, that precisely fit the observations. As we work through his brilliant analysis, you will see the series of problems that he solved.

To derive the earth's orbit he began by considering the moments when the sun, earth and Mars are essentially in a straight line. After 687 days, as Copernicus had found, Mars would return to the same place in its orbit. Of course, the earth at that time would not be at the same place in its orbit as when the first observation was made. Nevertheless, as Fig. 7.4 indicates, the directions of the earth as seen from the sun and from Mars would be known. The crossing point of the sight-lines from the sun and from Mars must be a point on the earth's orbit. By working with several groups of observations made 687 days apart, Kepler was able to determine fairly accurately the shape of the earth's orbit.

The orbit Kepler found for the earth appeared to be almost a circle, with the sun a bit off center. Kepler also knew, as you have read in Sec. 5.7, that the earth moves around the sun fastest during December and January, and slowest during June and July. Now he had an orbit and timetable for the earth's motion. In Experiment 15 you made a similar plot of the earth's orbit.

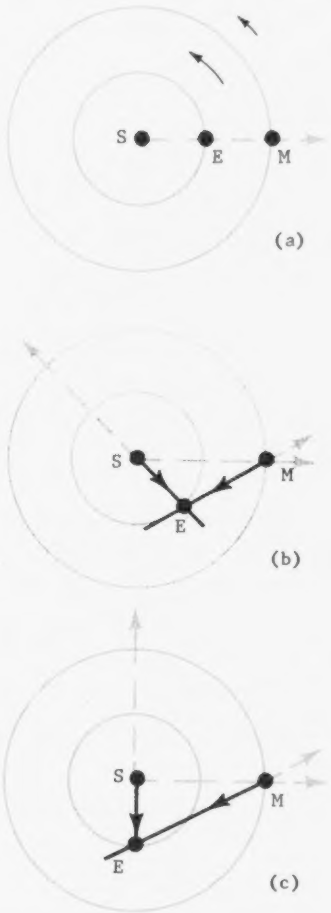


Fig. 7.4 How Kepler determined approximately the shape of the earth's orbit. Initially, (a) Mars is opposite the sun. After 687 days, (b) Mars has returned to the same place in its orbit, but the earth is almost 45° short of being at its initial position. After Mars makes one more cycle, (c) the earth lags by about 90°. Since the directions from the earth to the sun and Mars are known, the directions of the earth as seen from the sun and Mars are also known. Where these pairs of sight-lines cross must be points on the earth's orbit.

With the orbit and timetable of the earth known, Kepler could reverse the analysis. He triangulated the positions of Mars when it was at the same place in its orbit. For this purpose he again used observations separated by one orbital period of Mars (687 days). Because this interval is somewhat less than two earth years, the earth is at different positions in its orbit at the two times (Fig. 7.5). Then the two directions from the earth toward Mars differ, and two sight-lines can be drawn from the earth's positions; where they cross is a point on the orbit of Mars. From such pairs of observations Kepler fixed points on the orbit of Mars. From a curve drawn through such points you can get fairly accurate values for the size and shape of Mars' orbit. Kepler saw at once that the orbit of Mars was not a circle around the sun. You will find the same result from Experiment 18.

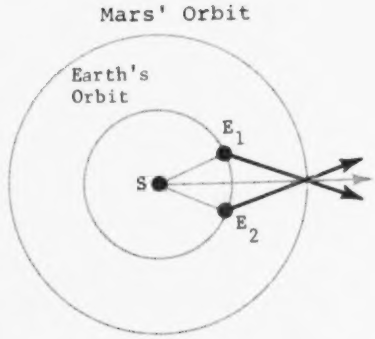


Fig. 7.5 How Kepler determined points on the orbit of Mars by triangulation.

Because Mars, like the earth, moves faster when nearer the sun, Kepler began to wonder why this occurred. Perhaps the sun exerted some force which pushed the planets along their orbits. Here we see the beginnings of a major change in interpretation. In the systems of Ptolemy and Copernicus, the sun was not a special object in a mechanical or dynamical sense. Even in the Copernican system each planet moved around its special point near the sun. No physical relation had been assumed, only a geometrical arrangement. Motions in the heavens had been considered as perpetual motions along circles. Now Kepler began to suspect that there was some physical interaction between the sun and the planets which caused the planets to move along their orbits.

While Kepler was studying how the speed of the planet changed along its orbit, he made an unexpected discovery: during equal time intervals a line drawn from the sun to the moving planet swept over equal areas. Figure 7.6 illustrates this for an orbit in which each pair of points is separated by equal time intervals. Between points A and B, the planet moves rapidly; between points G and H it moves slowly. Yet the areas swept over by the line from the sun to the planet are equal. Although Kepler discovered this Law of Areas before he discovered the exact shape of the orbits, it has become known as Kepler's second law. In its general form the Law of Areas states: the line from the sun to the moving planet sweeps over areas that are proportional to the time intervals.

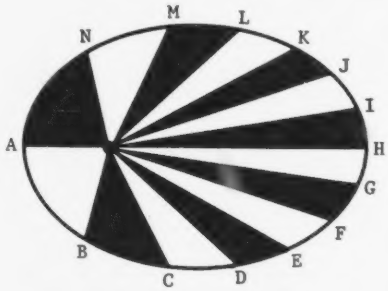
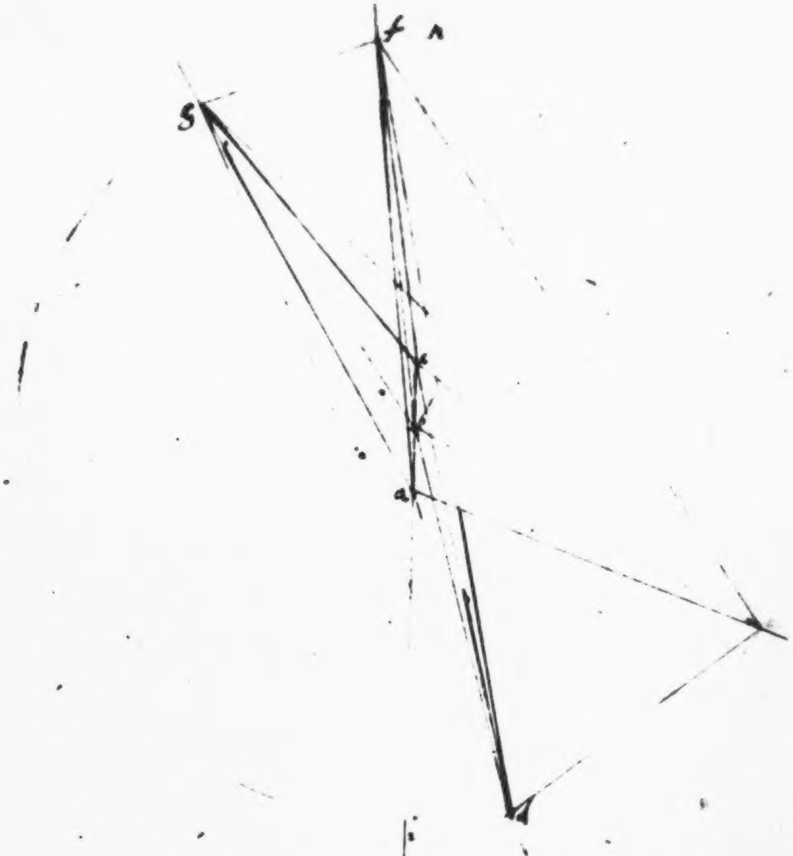


Fig. 7.6 Kepler's second law. A planet moves along its orbit at a rate such that the line from the sun to the planet sweeps over areas which are proportional to the time intervals.

Perhaps you are surprised that the first general law about the motions of the planets is concerned with the areas swept over by the line from the sun to the planet. After we have considered circles, eccentric circles, epicycles and equants to describe the motion, we come upon a quite unexpected property, the area swept over per unit time, as the first property of the orbital motion to remain constant. As we shall see in Chapter 8, this major law of nature applies to all orbits in the solar system and also to double stars. Perhaps you can sympathize with Kepler, who wrote that he was in ecstasy when, after great labor and ingenuity, he finally found this law. At last the problem was beginning to crack.



B Centrum orbitae ♂. A corpus solis. C centrum aequalitatis Martis GFHEDI orbita Martis. HAK Linea apogaei.

F Situs Martis anno 87 in [illegible]
G Situs Martis anno 91 in 20 [illegible]
D Situs Martis anno 93 in 17 [illegible]
E Situs Martis anno 95 in 18 [illegible]

F. 147. 26. 0.
G. 238. 36. 24
91 10 24

G 238. 36. 24
D 314. 6. 15
75. 29 51

D 314 6. 15
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EAF.	128. 10. 3.	ECF	143. 36. 8
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Summa CGA, ADC . 11. 21. 15
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Summa CGA, AEC. 19. 14. 45.
Summa CFA, ADC 7. 32. 35.

Excessus ~~CGA~~ super ~~CFA~~ CGA, ADC: — 7. 53. 30.
Excessus ~~CFA, CE, AEC~~ — 3. 48. 40.
[illegible] DAEC super CFA

[illegible] certa et data sunt. Jam oportet Apogaeum H sic ordinare, ut anguli quos faciunt lineae FB, GB, DB, EB, sint [illegible], quod erit indicium, illas [illegible] in [illegible]. Positis lineis [illegible] gala est ut necessarius: [illegible] [illegible] [illegible] [illegible]

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FCG 94. 59. 4
GCD 64. 8. 36
FCD 159. 7. 40

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7 32 35.

DAE 65. 9. 42
DCE. 57. 16. 12
7. 52 30.

GAE 91. 10. 24
GCF 94. 59. 4
3. 48. 40

As we shall see, Kepler's other labors would have been of little use without this basic discovery, even though it was an empirical discovery—without any hint why this law should be. The Law of Areas establishes the rate at which Mars (or any other planet or comet) moves at a particular point of its orbit. But to use this for making predictions of positions, viewed from the sun or from the earth, Kepler needed also to know the precise size and shape of Mars' orbit.

Q3 What types of observations did Kepler use for his study of Mars?

Q4 What were the new problems that Kepler had to solve?

Q5 What important simplifying assumption about planetary orbits was added by Kepler?

Q6 State Kepler's Law of Areas.

Q7 Summarize the steps Kepler used to determine the orbit of the earth.

Q8 Describe the velocity changes of a planet as it goes around the sun in an elliptical orbit. (See next section.)

7.3 Kepler's Law of Elliptical Orbits. By using the analysis we have described and illustrated in Fig. 7.5, p. 53, Kepler established some points on the orbit of Mars. But what sort of a path was this? How could he describe it? As Kepler said, "The conclusion is quite simply that the planet's path is not a circle—it curves inward on both sides and outward again at opposite ends. Such a curve is called an oval." But what kind of oval?

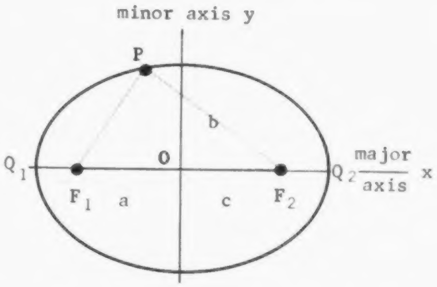


Fig. 7.7 An ellipse showing the semi-major axis a, the semi-minor axis b, and the two foci F_1 and F_2. The shape of an ellipse is described by its eccentricity, e, where $e = FO/OQ$, or $e = c/a$. (7.1)

Many different closed curves can be called ovals. Kepler thought for a time that the orbit was egg-shaped. Because such a shape did not agree with Kepler's ideas of physical interaction between the sun and the planet, he rejected the possibility that the orbit was egg-shaped. Kepler concluded that there must be some better way to describe the orbit and that he could find it. For many months, during which he often was ill and poverty-stricken, Kepler struggled with the question incessantly. Finally he was able to show that the orbit was a simple curve which had been studied in detail by the Greeks two thousand years before. The curve is called an ellipse. It is the shape you see when you view a circle at a slant.

In any ellipse the sum of the distances from the two foci to a point on the curve equals the length of the major axis, or $(F_1P + F_2P) = 2a$.

This property of ellipses allows us to draw them by using a loop of string around two tacks at the foci. Should the length of string tied into the loop have a total length of $(2a + 2c)$?

As Fig. 7.8 shows, ellipses can differ greatly in shape. They also have many interesting properties. For example, you can draw an ellipse by looping a piece of string around two thumb tacks pushed into a drawing board or cardboard at points F_1 and F_2 as shown in Fig. 7.7. Then, with a pencil point pull the loop taut and run the pencil once around the loop. You will have drawn an ellipse. (If the two thumb tacks had been together, what curve would you have drawn? What results do you get as you separate the two tacks more?)

A page from Kepler's notebooks.

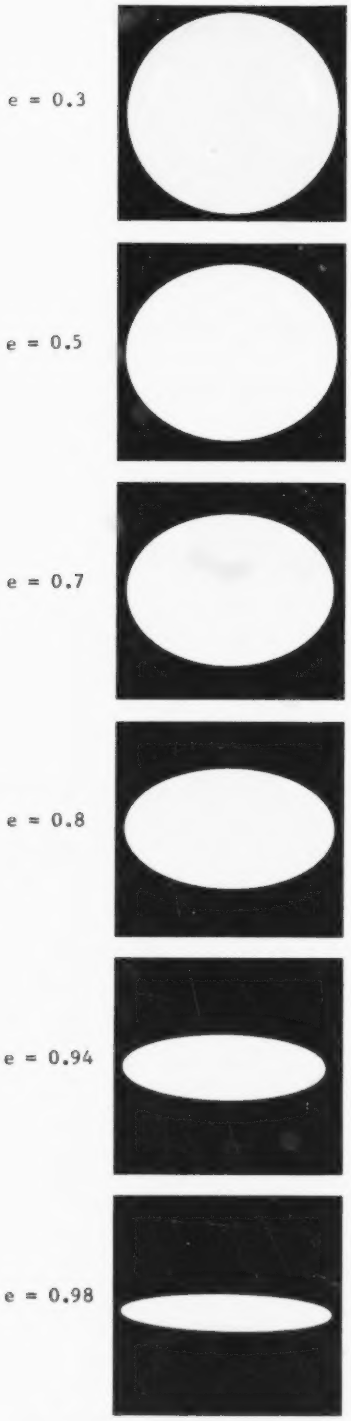


Fig. 7.8 Ellipses of different eccentricities. A saucer was photographed at various angles.

Each of the important points F_1 and F_2 is called a focus of the ellipse.

What Kepler discovered was not merely that the orbit of Mars is an ellipse—a remarkable enough discovery in itself—but also that the sun is at one focus. (The other focus is empty.) Kepler stated these results in his Law of Elliptical Orbits: the planets move in orbits which are ellipses and have the sun at one focus. That this is called Kepler's first law, although discovered after the Law of Areas, is an historical accident.

The long or major axis of an ellipse has a length 2a. The short or minor axis, perpendicular to the major axis, has a length 2b. The point O midway between the two foci, F_1 and F_2, is called the center, and the distance between the foci is called 2c. Thus the distance from the center to either focus is c.

The distance from any point P on the ellipse to the two foci can easily be found. Imagine that the loop is pulled out until your pencil is at the extreme point Q_2. Here the distance F_1Q_2 is $(a + c)$. At the other extreme point Q_1 the distance from F_2 is also $(a + c)$. When we subtract the distance 2c between the foci, the remainder is

$$F_1P + F_2P = 2a.$$

For astronomical orbits the distance a is called the mean distance of a point P from one focus (the sun).

As you probably discovered, the shape of an ellipse depends upon the distance between the foci. For that reason the shape of an ellipse is described by the ratio c/a, which is called the eccentricity of the ellipse and is denoted by e. Thus $e = c/a$. For a circle, which is an extreme form of an ellipse, the foci are together. Then the distance between foci is zero and the eccentricity is also zero. Other ellipses have eccentricities ranging between 0 and 1.

If Fig. 7.7 were to represent a planetary orbit, the sun would be at one focus, say F_1, with no object at the other focus. The planet would be nearest the sun when it reached point Q_1, and farthest from the sun at point Q_2. The point nearest the sun is called the perihelion point and the point farthest from the sun is called the aphelion point. The distances of these two points from the sun are called the perihelion distance and the aphelion distance respectively.

An example will show how these properties of an ellipse can be used to provide new interesting information. For the planet Mercury the perihelion distance (Q_1F_1 in Fig. 7.7)

The terms perihelion and aphelion come from the Greek, in which helios is the sun, peri means nearest, and apo means farthest. What other words do you know in which the prefixes peri and apo, or ap, have similar meanings?

has been found to be about 45.8×10^6 kilometers while the aphelion distance F_1Q_2 is about 70.0×10^6 kilometers. What is the eccentricity of the orbit of Mercury?

$$e = c/a$$

$$a = \frac{(Q_1F_1 + F_1Q_2)}{2}, \text{ or}$$

$$= \frac{45.8 \times 10^6 \text{ km} + 70.0 \times 10^6 \text{ km}}{2}$$

$$= 57.9 \times 10^6 \text{ km}$$

$$c = (\text{mean distance} - \text{perihelion distance})$$

$$= (OQ_1 - QF_1)$$

$$= 57.9 \times 10^6 \text{ km} - 45.8 \times 10^6 \text{ km}$$

$$= 12.1 \times 10^6 \text{ km}.$$

$$\text{Then } e = \frac{12.1 \times 10^6 \text{ km}}{57.9 \times 10^6 \text{ km}} = 0.21 .$$

As Table 7.1 shows, the orbit of Mars has the largest eccentricity of all the orbits that Kepler could study; those of Venus, earth, Mars, Jupiter and Saturn. Had he studied any planet other than Mars he might never have noticed that the orbit was an ellipse. Even for the orbit of Mars, the difference between the elliptical orbit and an off-center circle is quite small. No wonder Kepler later wrote that "Mars alone enables us to penetrate the secrets of astronomy."

Table 7.1 The Eccentricities of Planetary Orbits

Planet	Orbital Eccentricity	Notes
Mercury	0.206	Too few observations for Kepler to study
Venus	0.007	Nearly circular orbit
Earth	0.017	Small eccentricity
Mars	0.093	Largest eccentricity among planets Kepler could study
Jupiter	0.048	Slow moving in the sky
Saturn	0.056	Slow moving in the sky
Uranus	0.047	Not discovered until 1781
Neptune	0.009	Not discovered until 1846
Pluto	0.249	Not discovered until 1930

The work of Kepler illustrates the enormous change in outlook in Europe that had begun well over two centuries before. Scientific thinkers gradually ceased trying to impose human forms and motivations upon nature. Instead, they were beginning to look for and theorize about mathematical simplicities and mechanical or other models. Kepler rejected the ancient idea that each planet had a "soul." Instead he, like Galileo, began to search for physical causes. Where Copernicus and Tycho were willing to settle for geometrical models by which planetary positions could be predicted, Kepler was one of the first to seek causes for the motions. This new desire for causal explanations marks the beginning of modern physical science.

Like Kepler, we must have faith that our observations represent some aspects of a reality that is more stable than the emotions, wishes and behavior of human beings. Like Plato and all scientists, we assume that nature is basically orderly, consistent and therefore understandable in a simple way. This faith has led scientists to devote themselves to careful and sometimes tedious quantitative investigations of nature. As we all know, great theoretical and technical gains have resulted. Kepler's work illustrates one of the scientific attitudes—to regard a wide variety of phenomena as better understood when they can be described by simple, preferably mathematical patterns.

After Kepler's initial joy over the discovery of the law of elliptical paths, he may have asked himself the question: why are the planetary orbits elliptical rather than some other geometrical shape? While we can understand Plato's desire for uniform circular motions, nature's insistence on the ellipse is a surprise.

In fact, there was no satisfactory answer to Kepler's question until Newton showed, almost eighty years later, that these elliptical orbits were required by a much more general law of nature. Let us accept Kepler's laws as rules that contain the observed facts about the motions of the planets. As empirical laws, they each summarize the verifiable data from observations of the motion of any one planet. The first law, which describes the paths of planets as elliptical around the sun, gives us all the possible positions of each planet. That law, however, does not tell us when a planet will be at any one particular position on its ellipse or how rapidly it will then be moving. The second law, the Law of Areas, describes how the speed changes as the distance from the sun changes, but does not involve the shape of the orbit. Clearly these two laws complement each other. With these two general laws, numbers for the size and shape of the orbit, and the date for one position, we can determine both the position and angular speed of a given planet at any time relative to the sun. Since we can also find where the earth is at the same instant, we can derive the position of the planet as seen from the earth.

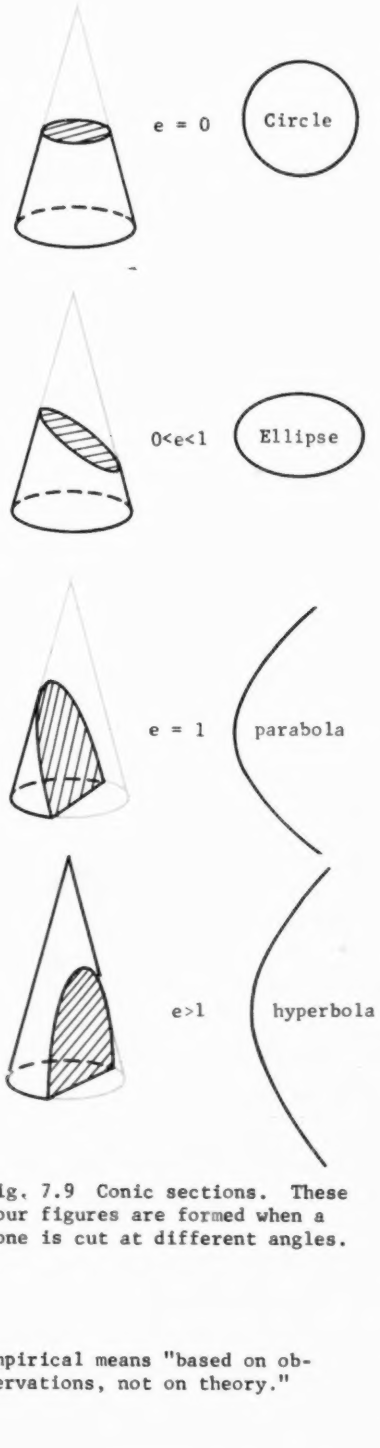


Fig. 7.9 Conic sections. These four figures are formed when a cone is cut at different angles.

Empirical means "based on observations, not on theory."

Q9 What can you do to a circle to have it appear as an ellipse?

Q10 Why was Kepler's decision to study the motion of Mars fortunate?

Q11 Summarize the steps Kepler used to determine the orbit of Mars.

7.4 Using the first two laws. Fig. 7.10 represents the elliptical path of a planet with the sun at one focus. By a short analysis we can determine the ratios of the speeds v_p and v_a of the planet when it is closest to the sun and farthest from the sun. As you note in Fig. 7.10, at these two points the velocity vectors are perpendicular to the radius. In Fig. 7.6, p. 53, the time intervals during which the planet moved from one marked point to the next are equal. In imagination let us make that time interval very small, Δt. Then the orbital speed becomes the instantaneous linear speed v. Also, as Fig. 7.11 suggests, the distances from the sun at the beginning and end of that interval are almost equal, so we may use R for both distances. Since we know that the area of any triangle is ½ base × altitude we may write the area of any such long thin triangle between the sun and a small section of the orbit as: area = $\frac{1}{2} Rv\Delta t$. But by Kepler's Law of Areas, when the time intervals Δt are equal the areas swept over by the radius are also equal. Thus we can equate the area at aphelion, R_a, to the area at perihelion, R_p, and have

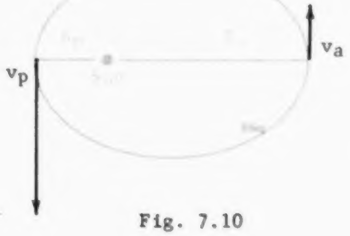


Fig. 7.10

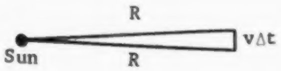


Fig. 7.11

$$\frac{1}{2} R_a v_a \Delta t = \frac{1}{2} R_p v_p \Delta t.$$

After cancelling out the common parts on both sides of the equation, we have

$$R_a v_a = R_p v_p. \qquad (7.2)$$

We can rearrange this equation to obtain the more interesting form:

$$v_a/v_p = R_p/R_a. \qquad (7.2a)$$

Eq. (7.2a) shows that the speeds at perihelion and aphelion are inversely proportional to the radii at these two points. If we return to the example about the orbit of Mercury, p. 57, we can find how the speeds at perihelion and aphelion compare:

SG 7.3

SG 7.5

$$v_a/v_p = R_p/R_a$$

$$v_a/v_p = 45.8 \times 10^6 \text{ km}/70.0 \times 10^6 \text{ km}$$

$$= 0.65.$$

The speed of Mercury at aphelion is only about 2/3 that at perihelion.

When Mercury is at perihelion at 0.31 A.U., its orbital speed is 58 km/sec. What is its orbital speed at aphelion at 0.47 A.U.?

Does Eq. (7.2a), derived for perihelion and aphelion positions, apply to any point on the orbit, or was some assumption hidden in our analysis? We must be careful not to fool ourselves, whether in words or in mathematical symbols. We did make an assumption: that the displacement, $v\Delta t$, taken as the altitude of the thin triangle, was perpendicular to the radial, sun-planet line. Therefore our results apply only to the two points in the orbit where the velocity vector is perpendicular to the radius. This condition occurs only at the perihelion and aphelion points.

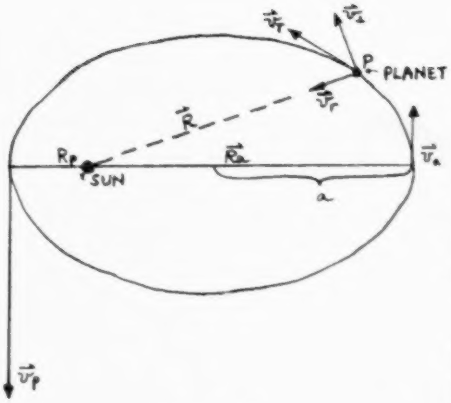


Fig. 7.12

The equation which takes the place of Eq. (7.2a) and holds for any two points along the orbit is $R_1(v_1)_\perp = R_2(v_2)_\perp$, where the $\perp$ means that we consider only the component of the velocity which is perpendicular to the sun-planet line. Can you see how you could use this knowledge to derive the velocity at any point on the orbit? Remember that the velocity vector is always tangent to the orbit. Figure 7.12 shows the relationships.

When we know the size and eccentricity of the elliptical orbit and apply Kepler's two simple laws, we can predict for past or future dates where the planet will be along its orbit. The elegance and simplicity of Kepler's two laws are impressive. Surely Ptolemy and Copernicus would have been amazed that the solution to the problem of planetary motions could be given by such short statements. But we must not forget that these laws were distilled from Copernicus' idea of a moving earth, the great labors and expense that went into Tycho's fine observations, and the imagination, devotion and often agony of Kepler's labors.

7.5 Kepler's Law of Periods. Kepler's first and second laws were published in 1609 in his book Astronomia Nova, or New Astronomy. But Kepler was dissatisfied because he had not yet found any relation between the motions of the different planets. So far, each planet seemed to have its own elliptical orbit and speeds, but there appeared to be no overall pattern relating all planets. Kepler, who had begun his career by trying to explain the number of planets and their spacing, was convinced that there must be some regularity, or rule, linking all the motions in the solar system. His conviction was so strong that he spent years examining many possible combinations of factors to find, by trial and error, a third law that would relate all the planetary orbits. His long search, almost an obsession, illustrates a belief that has run through the whole history of science: that nature is simple, uniform and understandable. This belief has always been a source of inspiration in science, often sustaining a man's spirit during periods of fruitless labor. For Kepler it made bearable a life of heartbreaking personal misfortunes, so that in 1619 he could write triumphantly in his Harmony of the World:

> ...after I had by unceasing toil through a long period of time, using the observations of Brahe, discovered the true distances of the orbits, at last, at last, the true relation...overcame by storm the shadows of my mind, with such fullness of agreement between my seventeen years' labor on the observations of Brahe and this present study of mine that I at first believed that I was dreaming....

Kepler's third law, the Law of Periods, also called the Harmonic Law, relates the periods of the planets to their mean distances from the sun. The period T is the time taken to go once completely around the orbit; while the mean distance is a. Specifically, the law states that: the squares of the periods of the planets are proportional to the cubes of their mean distances from the sun. In the short form of algebra, this is,

SG 7.1

SG 7.2

SG 7.3

$$T^2 = ka^3, \text{ or } T^2/a^3 = k. \qquad (7.3)$$

For the earth, T is one year. The mean distance of the earth from the sun, a, is one astronomical unit, A.U. Then by Eq. (7.3) we have

$$(1 \text{ year})^2 = k(1 \text{ A.U.})^3, \text{ or } k = 1 \text{ yr}^2/\text{A.U.}^3.$$

Because this relation applies to all the planets, we can use it to find the period or mean distance of any planet when we know either of the quantities. Thus Kepler's third law, the Law of Periods, establishes a beautifully simple relation among the planetary orbits.

We know from observation that the orbital period T of Jupiter is about 12 years. What value for a, the mean distance from the sun, is predicted on the basis of Kepler's third law?

Solution:

$$\begin{aligned} a_j^3 &= T_j^2/k \\ &= 144\ (\text{yr}^2)/1\ (\text{yr}^2/\text{A.U.}^3) \\ &= 144\ \text{A.U.}^3 \\ a_j &= \sqrt[3]{144\ \text{A.U.}^3} \\ &= 5.2\ \text{A.U.} \end{aligned}$$

Kepler's three laws are so simple that their great power may be overlooked. When they are combined with his discovery that each planet moves in a plane passing through the sun, they let us derive the past and future history of each planet's motion from only six quantities, known as the orbital elements. Two of the elements are the size and shape of the orbit in its plane, three other elements are angles that orient the planet's orbit in its plane and relate the plane of the planet to that of the earth's orbit, while the sixth element tells where in the orbit the planet was on a certain date. These elements are explained more fully in optional Experiment 21 on the orbit of Halley's Comet.

It is astonishing that the past and future positions of each planet can be derived in a simpler and more precise way than through the multitude of geometrical devices on which all other planetary theories depended, whether those of Ptolemy, Copernicus or Tycho. With different assumptions and procedures Kepler had at last solved the astronomical problem on which so many great men had worked so hard for centuries. Although he had to abandon the geometrical devices of the Copernican system, Kepler did depend on the Copernican model. None of the earth-centered models could have led to Kepler's three laws.

In 1627, after many troubles with publishers, Kepler finally published the Rudolphine Tables based on Tycho's observations. In these tables Kepler combined Tycho's observations and Kepler's laws in a way that permitted accurate calculations of planetary positions for any times, whether in the past or future. These Rudolphine Tables remained useful for a century until telescopic observations of greater precision replaced Tycho's observations.

These tables were also important for a quite different reason. In them Kepler pioneered in the use of logarithms and included a long section, practically a textbook, on the nature of logarithms and their use for calculations. Kepler had realized that logarithms, first described in 1614 by Napier in Scotland, would be very useful in speeding up the tedious arithmetic required for the derivation of planetary positions.

We honor Kepler for his astronomical and mathematical achievements, but these were only a few of the accomplishments of this great man. As soon as Kepler learned of the development of the telescope, he spent most of a year making careful studies of how the images were formed. These he published in a book titled Dioptrice, which became the standard work on optics for many years. Like Tycho, who was much impressed by the new star of 1572, Kepler similarly observed and wrote about the new stars of 1600 and 1604. His observations and interpretations added to the impact of Tycho's similar observations in 1572 that changes did occur in the starry sky. In addition to a number of important books on mathematical and astronomical problems, Kepler wrote a popular and widely read description of the Copernican system as modified by his own discoveries. This added to the growing interest in and acceptance of the sun-centered model of the planetary system.

Q12 State Kepler's third law, the Law of Periods. Why is it useful?

7.6 The new concept of physical law. One general feature of Kepler's life-long work has had a profound effect on the development of all the physical sciences. When Kepler began his studies he still accepted Plato's assumptions about the importance of geometric models and Aristotle's emphasis on natural place to explain motion. But at the end he stated mathematical laws describing how planets moved, and even attempted to explain these motions in terms of physical forces. His successful statement of empirical laws in mathematical form helped to establish the equation as the normal form of laws in physical science. Thus he contributed to a new way of considering observations and stating conclusions.

More than anyone before him, Kepler expected an acceptable theory to agree with precise and quantitative observation. In Kepler's system the planets no longer were considered to move in their orbits because they had some divine nature or influence, or because they had spherical shapes which served as self-evident explanation for their circular motions. Rather, Kepler tried to back up his mathematical descriptions with physical mechanisms. In fact, he was the first to look for a universal physical law based on terrestrial phenomena to describe the whole universe in quantitative detail. In an early letter he expressed his guiding thought:

See "A Search for Life on Earth" in Project Physics Reader 2.

See Galileo's "The Starry Messenger" in Project Physics Reader 2.

> I am much occupied with the investigation of the physical causes. My aim in this is to show that the celestial machine is to be likened not to a divine organism but rather to a clockwork...insofar as nearly all the manifold movements are carried out by means of a single, quite simple magnetic force, as in the case of a clockwork, all motions [are caused] by a simple weight. Moreover, I show how this physical conception is to be presented through calculation and geometry. [Letter to Herwart, 1605.]

The world as a celestial machine driven by a single force, in the image of a clockwork—this was indeed a prophetic goal! Stimulated by William Gilbert's work on magnetism a few years earlier, Kepler could imagine magnetic forces from the sun driving the planets in their orbits. This was a promising and reasonable hypothesis. As it turned out, the fundamental idea that a single kind of force controls the motions of all the planets was correct; but the force is not magnetism and it is not needed to keep the planets moving.

Kepler's statements of empirical laws remind us of Galileo's suggestion, made at about the same time, that we deal first with the how and then with the why of motion in free fall. A half century later Newton used gravitational force to tie together Kepler's three planetary laws for a heliocentric system with the laws of terrestrial mechanics in a magnificent synthesis (Chapter 8).

Q13 To what did Kepler wish to compare the "celestial machine"?

Q14 Why is Kepler's reference to a "clockwork" model significant?

7.7 Galileo's viewpoint. One of the scientists with whom Kepler corresponded about the latest scientific developments was Galileo. Kepler's contributions to planetary theory were mainly his empirical laws based on the observations of Tycho. Galileo contributed to both theory and observation. As Chapters 2 and 3 reported, Galileo's theory was based on observations of bodies moving on the earth's surface. His development of the new science of mechanics contradicted the assumptions on which Aristotle's physics and interpretation of the heavens had been based. Through his books and speeches Galileo triggered wide discussion about the differences or similarities of earth and heaven. Outside of scientific circles, as far away as England, the poet John Milton wrote, some years after his visit to Galileo in 1638:

> ...What if earth
> Be but the shadow of Heaven, and things therein
> Each to the other like, more than on earth is thought?

[Paradise Lost, Book V, line 574, 1667.]

Galileo challenged the ancient interpretations of experience. As you saw earlier, he focused attention on new concepts: time and distance, velocity and acceleration, forces and matter in contrast to the Aristotelian qualities or essences, ultimate causes, and geometrical models. In Galileo's study of falling bodies he insisted on fitting the concepts and conclusions to the observed facts. By expressing his results in concise mathematical form, Galileo paralleled the new style being used by Kepler.

The sharp break between Galileo and most other scientists of the time arose from the kind of question he asked. To his opponents, many of Galileo's problems seemed trivial. What was important about watching pendulums swing or rolling balls down inclines when philosophical problems needed clarification? Certainly his procedures for studying the world seemed peculiar, even fantastic, and his statements were strange and his manner haughty.

Although Kepler and Galileo lived at the same time, their lives were quite different. Kepler was struggling to find simplicity among complex astronomical phenomena. He was enchanted by the mysteries of astrology; he lived a hand-to-mouth existence under stingy patrons, and was driven from city to city by the religious wars of the time. Few people, other than a handful of friends and correspondents, knew of or cared about his studies and results.

Galileo's situation was different. He wrote his numerous papers and books in Italian, which could be read by many people who did not read scholarly Latin. These publications were the work of a superb partisan and publicist. Galileo wanted many to know of his studies and to accept the Copernican theory. He took the argument far beyond a small group of scholars out to the nobles, civic leaders, and religious dignitaries. His reports and arguments, including often bitter ridicule of individuals or ideas, became the subject of dinner-table conversations. In his efforts to inform and persuade he stirred up the ridicule and even violence often poured upon those who have new ideas. In the world of art similar receptions were given initially to Manet and Giacometti, and in music to Beethoven, Stravinsky and Schönberg.

7.8 The telescopic evidence. Like Kepler, Galileo was a Copernican among Ptolemaeans who believed that the heavens were eternal and could not change. Hence, Galileo was interested in the sudden appearance in 1604 of a new star, one of those observed by Kepler. Where there had been nothing visible in the sky, there was now a brilliant star. Galileo, like Tycho and Kepler, realized that such changes in the starry sky conflicted with the old idea that the stars could not change. Furthermore, this new star awakened in Galileo an interest in astronomy which lasted throughout his life.

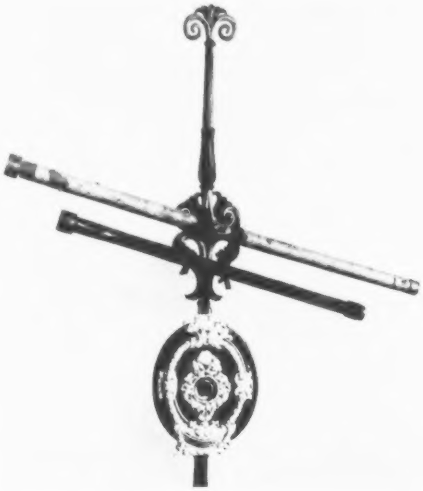


Fig. 7.13 Two of Galileo's telescopes displayed in Florence.

Consequently, Galileo was ready to react to the news he received four or five years later that a Dutchman "had constructed a spy glass by means of which visible objects, though very distant from the eye of the observer, were distinctly seen as if nearby." Galileo quickly worked out the optical principles involved, and set to work to grind the lenses and build such an instrument himself. His first telescope made objects appear three times closer than when seen with the naked eye. Then he constructed a second and a third telescope. Reporting on his third telescope, Galileo wrote:

> Finally, sparing neither labor nor expense, I succeeded in constructing for myself so excellent an instrument that objects seen by means of it appeared nearly one thousand times larger and over thirty times closer than when regarded with our natural vision" (Fig. 7.13).

Galileo meant that the area of the object was nearly 1000 times larger. The linear magnification was over 30 times.

What would you do if you were handed "so excellent an instrument"? Like the men of Galileo's time, you probably would put it to practical uses. "It would be superfluous," Galileo agreed,

> to enumerate the number and importance of the advantages of such an instrument at sea as well as on land. But forsaking terrestrial observations, I turned to celestial ones, and first I saw the moon from as near at hand as if it were scarcely two terrestrial radii away. After that I observed often with wondering delight both the planets and the fixed stars....

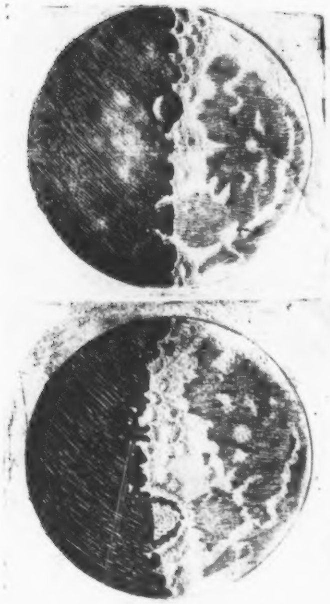


Fig. 7.14 Two of Galileo's early drawings of the moon. (From Galileo's Sidereus Nuncius, which is often translated as The Sidereal Messenger or as The Starry Messenger.)

What, then, were the findings that Galileo made with his telescope? In the period of a few short weeks in 1609 and 1610 he made several discoveries, each of which is of first rank.

First, Galileo pointed his telescope at the moon. What he saw led him to the conviction that

> ...the surface of the moon is not smooth, uniform, and precisely spherical as a great number of philosophers believe it [and the other heavenly bodies] to be, but is uneven, rough, and full of cavities and prominences, being not unlike the face of the earth, relieved by chains of mountains, and deep valleys. [See Fig. 7.14.]

Galileo did not stop with that simple observation, which was contrary to the Aristotelian idea of heavenly perfection. He supported his conclusions with several kinds of observations, including quantitative evidence. For instance, he worked out a method for determining the height of a mountain on the moon from the shadow it casts. His value of about four miles for the height of some lunar mountains is not far from modern results.

Next he looked at the stars. To the naked eye the Milky Way had seemed to be a continuous blotchy band of light; through the telescope it was seen to consist of thousands of faint stars. Wherever Galileo pointed his telescope in the sky he saw many more stars than could be seen with the unaided eye. This observation was contrary to the old argument that the stars were created to provide light so men could see at night. If that were the explanation, there should not be stars invisible to the naked eye—but Galileo found thousands.

After his observations of the moon and the fixed stars, Galileo turned his attention to the discovery which in his opinion

> ...deserves to be considered the most important of all —the disclosure of four PLANETS never seen from the creation of the world up to our own time; together with the occasion of my having discovered and studied them,

> their arrangements, and the observations made of their movements during the past two months.

He is here referring to his discovery of four (of the twelve now known) satellites which orbit about Jupiter (Fig. 7.15). Here, before his eyes, was a miniature solar system with its own center of revolution around Jupiter. This was directly opposed to the Aristotelian notion that the earth was at the center of the universe and could be the only center of revolution.

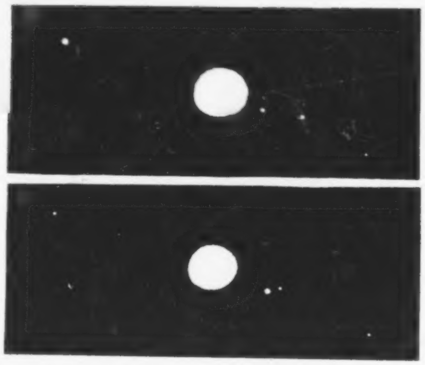


Fig. 7.15 Jupiter and its four brightest satellites. The lower photograph was taken 3 hours later than the upper one. (Photographed at Yerkes Observatory.)

The manner in which Galileo discovered Jupiter's "planets" is a tribute to his ability as an observer. Each clear evening during this period he was discovering dozens if not hundreds of new stars never before seen by man. When looking in the vicinity of Jupiter on the evening of January 7, 1610, he noticed "...that beside the planet there were three starlets, small indeed, but very bright. Though I believe them to be among the host of fixed stars, they aroused my curiosity somewhat by appearing to lie in an exact straight line..." In his notebook he made a sketch similar to that shown in the top line Fig. 7.16. When he saw them again the following night, he saw that they had changed position with reference to Jupiter. Each clear evening for weeks he observed that planet and its roving "starlets" and recorded their positions in drawings. Within days he had concluded that there were four "starlets" and that they were indeed satellites of Jupiter. He continued his observations until he was able to estimate the periods of their revolutions around Jupiter.

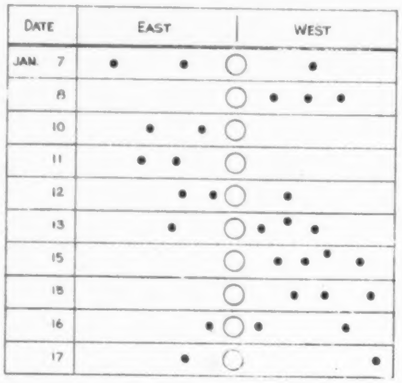


Fig. 7.16 Galileo observed and recorded the relative position of Jupiter's brightest satellites 64 times between January 7 and March 2, 1610. The sketches shown here are similar to Galileo's first ten recorded observations which he published in the first edition of his book Sidereus Nuncius, The Starry Messenger.

Of all of Galileo's discoveries, that of the satellites of Jupiter caused the most stir. His book, The Starry Messenger, was an immediate success, and copies were sold as fast as they could be printed. For Galileo the result was a great demand for telescopes and great public fame.

Galileo continued to use his telescope with remarkable results. By projecting an image of the sun on a screen, he observed sunspots. This was additional evidence that the sun, like the moon, was not perfect in the Aristotelian sense: it was disfigured rather then even and smooth. From his observation that the sunspots moved across the disk of the sun in a regular pattern, he concluded that the sun rotated with a period of about 27 days.

If there are four bright satellites moving about Jupiter, why could Galileo sometimes see only two or three? What conclusions can you draw from the observation (see Figs. 7.15 and 7.16) that they lie nearly along a straight line?

He also found that Venus showed all phases, just as the moon does (Fig. 7.17). Therefore Venus must move completely around the sun as Copernicus and Tycho had believed, rather than be always between the earth and sun as the Ptolemaic astronomers assumed (see again Fig. 5.15). Saturn seemed

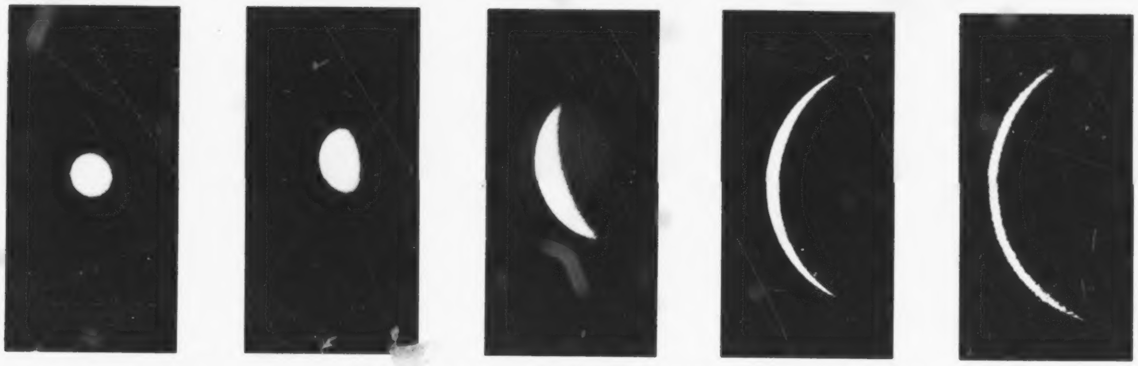


Fig. 7.17 Photographs of Venus at various phases taken with a constant magnification.

to carry bulges around its equator (Fig. 7.18), but Galileo's telescopes were not strong enough to show that they were rings. With his telescopes he collected an impressive array of new information about the heavens—and all of it seemed to contradict the basic assumptions of the Ptolemaic world scheme.

Q15 Could Galileo's observation of all phases of Venus support the heliocentric theory, the Tychonic system or Ptolemy's system?

Q16 In what ways did the invention of the telescope provide new evidence for the heliocentric theory?

Q17 What significance did observations of Jupiter have in the development of Galileo's ideas?

7.9 Galileo's arguments. To Galileo these observations supported his belief in the heliocentric Copernican system, but they were not the cause of them. In his great work, the Dialogue Concerning the Two Chief World Systems (1632), he stressed arguments based on assumptions as much as on observations. The observed motions of planets alone, says Galileo, do not decide uniquely between a heliocentric and a geocentric hypothesis, "for the same phenomena would result from either hypothesis." But Galileo accepted the earth's motion as real because it seemed to him simpler and more pleasing. Elsewhere in this course you will find other cases where a scientist accepted or rejected an idea for reasons arising from his strong belief in particular assumptions.

In the Dialogue Galileo presents his arguments in a systematic and lively way. The Dialogue, like the Discourses Concerning Two New Sciences, which are mentioned in Chapter 2, is in the form of a discussion between three learned men. Salviati, the voice of Galileo, wins most of the arguments. His antagonist is Simplicius, an Aristotelian who speaks for and defends the Ptolemaic system. The third member, Sagredo, represents the objective and intelligent citizen not yet committed to either system. However, Sagredo's role is written so that he usually accepts most of Galileo's arguments.

A small eighteenth-century reflecting telescope, now in the Harvard University collection.

Galileo's arguments in favor of the Copernican system as set forth in the Dialogue Concerning the Two Chief World Systems were mostly those given by Copernicus (see Chapter 6), and Galileo made no use of Kepler's laws. However, Galileo had new evidence from his own observations. After deriving the periods of Jupiter's four moons or satellites, Galileo found that the larger the orbit of the satellite, the longer was the period of revolution. Copernicus had already found that the periods of the planets increased with their average distances from the sun. (Kepler's Third Law stated the relation for the planets in detailed quantitative form.) Now Jupiter's satellite system showed a similar pattern. These new patterns of regularities would soon replace the old assumptions of Plato, Aristotle and Ptolemy.

The Dialogue Concerning the Two Chief World Systems relies upon Copernican arguments, Galilean observations and reasonableness to attack the basic assumptions of the geocentric model. Finally, Simplicius, seemingly in desperation, dismisses all of Galileo's arguments with a characteristic counterargument:

> ...with respect to the power of the Mover, which is infinite, it is just as easy to move the universe as the earth, or for that matter a straw.

But to this, Galileo makes a very interesting reply; notice how he quotes Aristotle against the Aristoteleans:

> ...what I have been saying was with regard not to the Mover, but only the movables....Giving our attention, then, to the movable bodies, and not questioning that it is a shorter and readier operation to move the earth than the universe, and paving attention to the many other simplifications and conveniences that follow from merely this one, it is much more probable that the diurnal motion belongs to the earth alone than to the rest of the universe excepting the earth. This is supported by a very true maxim of Aristotle's which teaches that..."it is pointless to use many to accomplish what may be done with fewer."

7.10 The opposition to Galileo. In his characteristic enthusiasm, Galileo thought that his telescopic discoveries would cause everyone to realize the absurdity of the assumptions that prevented a general acceptance of the Copernican system. But men can believe only what they are ready to believe. In their fight against the new Copernicans, the followers of Aristotle were convinced that they were surely sticking to facts and that the heliocentric theory was obviously false and in contradiction with both observation and common sense. The evidences of the telescope could be due to distortions. After all, glass lenses change the path of light rays; and even if telescopes seemed to work for terrestrial observation, nobody could be sure they worked equally well when pointed at these vastly more distant celestial objects.

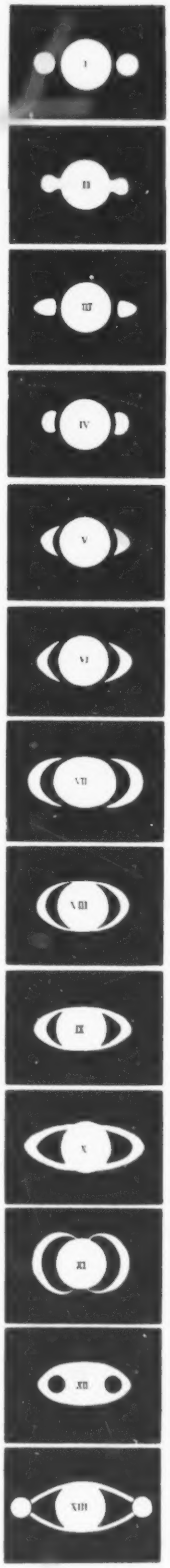


Fig. 7.18 Sketches of Saturn made from telescopic observations during the seventeenth century.

Consider each of Galileo's telescopic observations separately to determine whether it strengthened or weakened the case for (a) the geocentric theory, and (b) the philosophical assumptions underlying the geocentric theory. Give particular attention to the observed phases of Venus. Could these observations of Venus be explained by the Tychonic system? Could the Ptolemaic system be modified to have Venus show all phases?

Furthermore, theological heresies were implied in the heliocentric view. Following Thomas Aquinas, the Scholastics had adopted the Aristotelian argument as the only correct basis for building any physical theory. The Aristotelians could not even consider the Copernican system as a possible theory without giving up many of their basic assumptions, as you read in Chapter 6. To do so would have required them to do what is humanly almost impossible: discard their common-sense ideas and seek new bases for their moral and theological doctrines. They would have to admit that the earth is not at the center of creation. Then perhaps the universe was not created especially for mankind. Is it any wonder that Galileo's arguments stirred up a storm of opposition?

Galileo's observations were belittled by the Scholastics. The Florentine astronomer, Francesco Sizzi (1611), argued why there could not, indeed must not be any satellites around Jupiter:

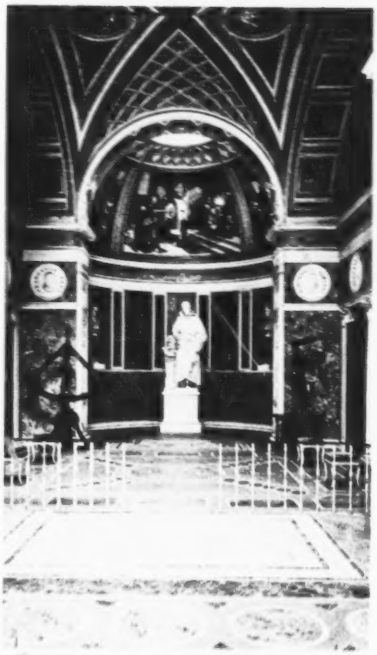
> There are seven windows in the head, two nostrils, two ears, two eyes and a mouth; so in the heavens there are two favorable stars, two unpropitious, two luminaries, and Mercury alone undecided and indifferent. From which and many other similar phenomena of nature such as the seven metals, etc., which it were tedious to enumerate, we gather that the number of planets is necessarily seven [including the sun and moon]....Besides, the Jews and other ancient nations, as well as modern Europeans, have adopted the division of the week into seven days, and have named them from the seven planets; now if we increase the number of planets, this whole system falls to the ground.... Moreover, the satellites are invisible to the naked eye and therefore can have no influence on the earth, and therefore would be useless, and therefore do not exist.

A year after his discoveries, Galileo wrote to Kepler:

> You are the first and almost the only person who, even after a but cursory investigation, has...given entire credit to my statements....What do you say of the leading philosophers here to whom I have offered a thousand times of my own accord to show my studies, but who with the lazy obstinacy of a serpent who has eaten his fill have never consented to look at the planets, or moon, or telescope?

7.11 Science and freedom. The political and personal tragedy that occurred to Galileo is described at length in many books. Here we shall only mention briefly some of the major events. Galileo was warned in 1616 by the Inquisition to cease teaching the Copernican theory as true, rather than as just one of several possible explanations, for it was now held contrary to Holy Scripture. At the same time Copernicus' book was placed on the Index Expurgatorius, and was suspended "until corrected." As we saw before, Copernicus had used Aristotelian doctrine to make his theory plausible. But Galileo had reached the new point of view where he urged acceptance of the heliocentric system on its own merits. While he was himself a devoutly religious man, he deliberately ruled out questions of religious faith and salvation from scientific discussions. This was a fundamental break with the past.

When, in 1623, Cardinal Barberini, formerly a dear friend of Galileo, was elected to be Pope Urban VIII, Galileo talked with him regarding the decree against the Copernican ideas. As a result of the discussion, Galileo considered it safe enough to write again on the controversial topic. In 1632, after making some required changes, Galileo obtained the necessary Papal consent to publish the work, *Dialogue Concerning the Two Chief World Systems*. This book presented most persuasively the Copernican view in a thinly disguised discussion of the relative merits of the Ptolemaic and Copernican systems. After the book's publication his opponents argued that Galileo seemed to have tried to get around the warning of 1616. Furthermore, Galileo's forthright and sometimes tactless behavior, and the Inquisition's need to demonstrate its power over suspected heretics, combined to mark him for punishment.



Over 200 years after his confinement in Rome, opinions had changed so that Galileo was honored as in the fresco "Galileo presenting his telescope to the Venetian Senate" by Luigi Sabatelli (1772-1850). The fresco is located in the Tribune of Galileo, Florence, which was assembled from 1841 to 1850.

Among the many factors in this complex story, we must remember that Galileo, while considering himself religiously faithful, had become a suspect of the Inquisition. In Galileo's letters of 1613 and 1615 he wrote that to him God's mind contains all the natural laws; consequently he held that the occasional glimpses of these laws which the human investigator may gain were proofs and direct revelations of the Deity, quite as valid and grand as those recorded in the Bible. "From the Divine Word, the Sacred Scripture and Nature did both alike proceed....Nor does God less admirably discover himself to us in Nature's actions than in the Scripture's sacred dictions." These opinions—held by many present-day scientists, and no longer regarded as being in conflict with theological doctrines—could, however, be regarded at Galileo's time as symptoms of pantheism. This was one of the heresies for which Galileo's contemporary, Giordano Bruno, had been burned at the stake in 1600. The Inquisition was alarmed by Galileo's contention that the Bible was not a certain source of knowledge for the teaching of natural science. Thus he quoted Cardinal Baronius' saying: "The Holy Spirit intended to teach us [in the Bible] how to go to heaven, not how the heavens go."

Though he was old and ailing, Galileo was called to Rome and confined for a few months. From the proceedings of Galileo's trial, of which parts are still secret, we learn that he was tried, threatened with torture, induced to make an elaborate formal confession of improper behavior and a denial of the Copernican theory. Finally he was sentenced to perpetual house arrest. According to a well-known legend, at the end of his confession Galileo muttered "eppur si muove"—"but it does move." None of Galileo's friends in Italy dared to defend him publicly. His book was placed on the Index where it remained, along with that of Copernicus and one of Kepler's, until 1835. Thus he was used as a warning to all men that the demand for spiritual conformity also required intellectual conformity.

But without intellectual freedom, science cannot flourish for long. Perhaps it is not a coincidence that for 200 years after Galileo, Italy, which had been the mother of outstanding men, produced hardly a single great scientist, while elsewhere in Europe they appeared in great numbers. Today scientists are acutely aware of this famous part of the story of planetary theories. Teachers and scientists in our time have had to face powerful enemies of open-minded inquiry and of free teaching. Today, as in Galileo's time, men who create or publicize new thoughts must be ready to stand up before other men who fear the open discussion of new ideas.

Plato knew that an authoritarian state is threatened by intellectual nonconformists and had recommended for them the well-known treatments: "reeducation, prison, or death." Recently, Russian geneticists have been required to reject well-established theories, not on grounds of persuasive new scientific evidence, but because of conflicts with political doctrines. Similarly, discussion of the theory of relatively was banned from textbooks in Nazi Germany because Einstein's Jewish heritage was said to invalidate his work for Germans. Another example of intolerance was the "Monkey Trial" held during 1925 in Tennessee, where the teaching of Darwin's theory of biological evolution was attacked because it conflicted with certain types of Biblical interpretation.

The warfare of authoritarianism against science, like the warfare of ignorance against knowledge, is still with us. Scientists take comfort from the verdict of history. Less than 50 years after Galileo's trial, Newton's great book, the Principia, brilliantly united the work of Copernicus, Kepler and Galileo with Newton's new statements of principles of mechanics. Thus the hard-won new laws and new views of science were established. What followed has been termed by historians The Age of Enlightenment.

SG 7.6

Q18 What major change in the interpretations of observations was illustrated by the work of both Kepler and Galileo?

Q19 What are some of the reasons that caused Galileo to be tried by the Inquisition?

Palomar Observatory houses the 200-inch Hale reflecting telescope. It is located on Palomar Mountain in southern California.

Study Guide

7.1 If a comet in an orbit around the sun has a mean distance of 20 A.U. from the sun, what will be its period?

7.2 A comet is found to have a period of 75 years.

- a) What will be its mean distance from the sun?
- b) If its orbital eccentricity is 0.90, what will be its least distance from the sun?
- c) What will be its velocity at aphelion compared to its velocity at perihelion?

7.3 What is the change between the earth's lowest speed in July when it is 1.02 A.U. from the sun and its greatest speed in January when it is 0.98 A.U. from the sun?

7.4 The mean distance of the planet Pluto from the sun is 39.6 A.U. What is the orbital period of Pluto?

7.5 The eccentricity of Pluto's orbit is 0.254. What will be the ratio of the minimum orbital speed to the maximum orbital speed of Pluto?

7.6 What are the current procedures by which the public is informed of new scientific theories? To what extent do these news media emphasize any clash of assumptions?

Chapter 8 The Unity of Earth and Sky — The Work of Newton

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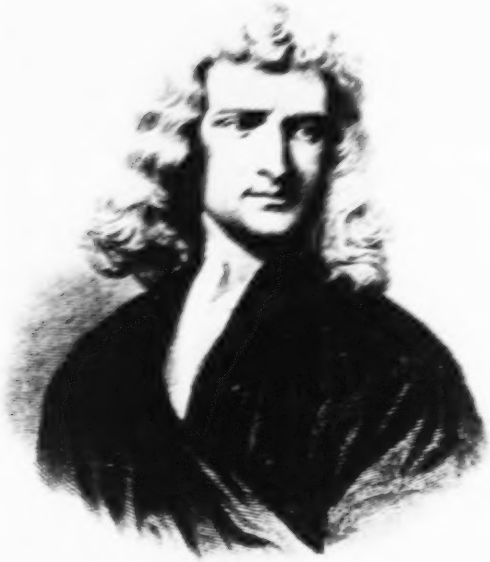


Is. Newton

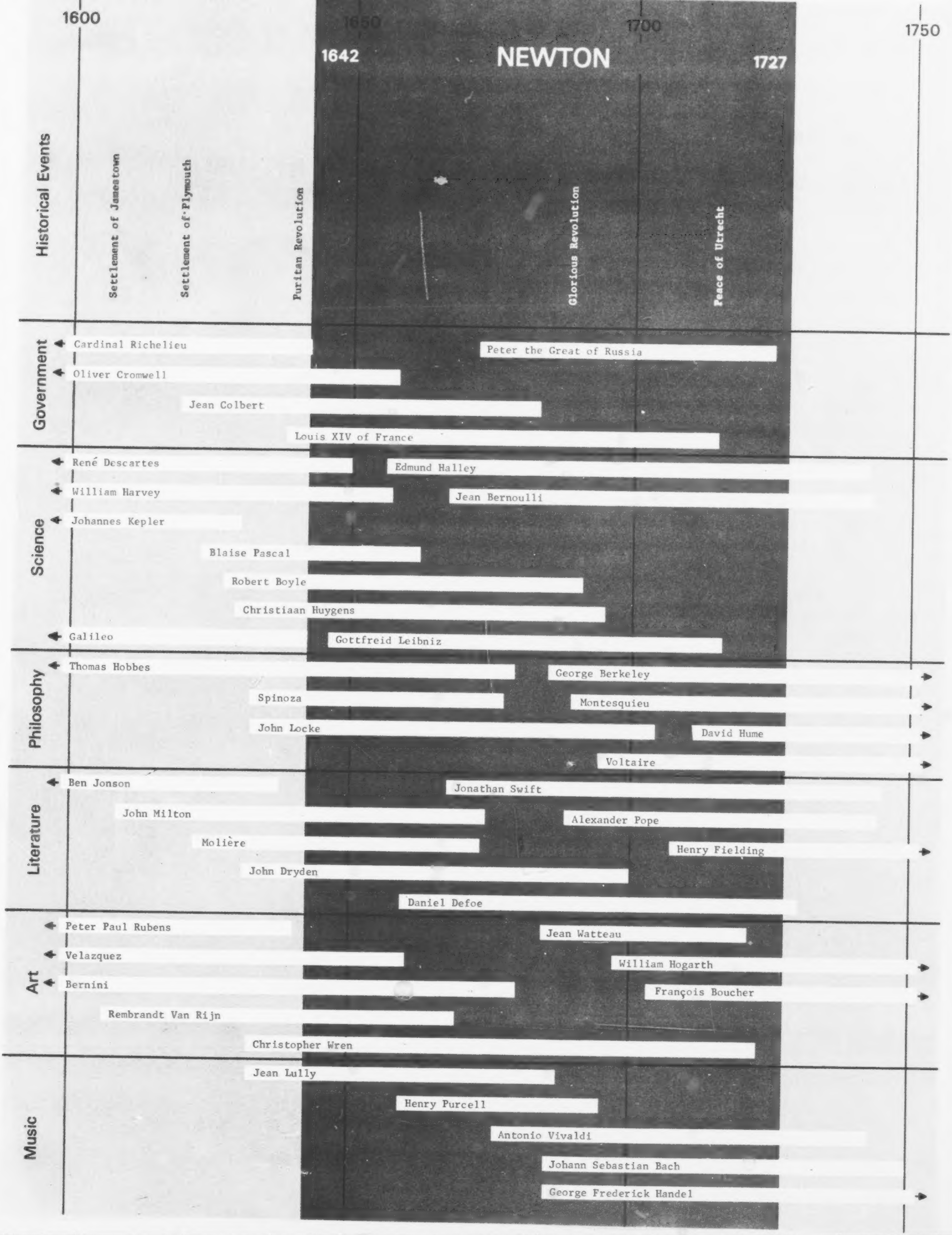
Introduction: Science in the seventeenth century. In the 44 years between the death of Galileo in 1642 and the publication of Newton's Principia in 1687, major changes occurred in the social organization of scientific studies. The "New Philosophy" of experimental science, applied by enthusiastic and imaginative men, was giving a wealth of new results. Because these men were beginning to work together, they formed scientific societies in Italy, England and France. One of the most famous is the Royal Society of London for Promoting Natural Knowledge, which was founded in 1660. Through these societies the scientific experimenters exchanged information, debated new ideas, published technical papers, and sometimes quarreled heartily. Each society sought support for its work, argued against the opponents of the new experimental activities and published studies in scientific journals, which were widely read. Through the societies scientific activities were becoming well-defined, strong and international.

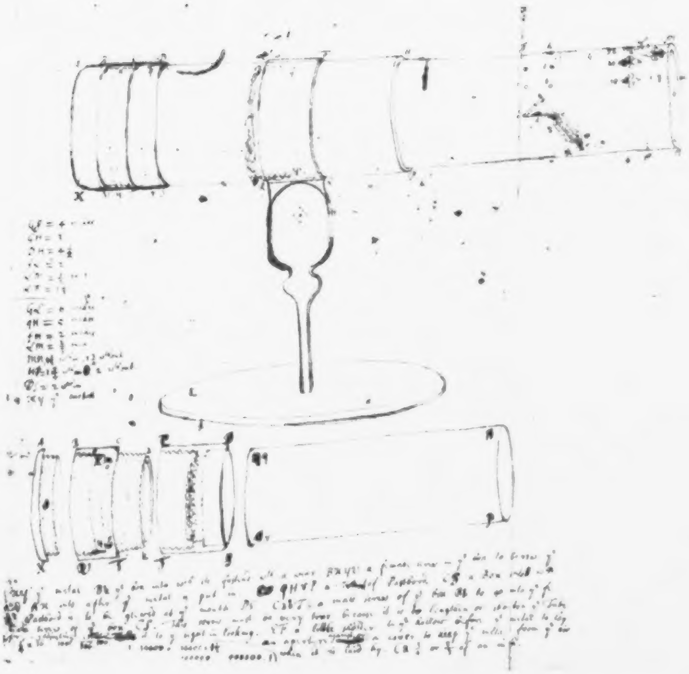
This development of scientific activities was part of the general cultural, political and economic changes occurring in the sixteenth and seventeenth centuries (see the chart). Both craftsmen and men of leisure and wealth became involved in scientific studies. Some sought the improvement of technological methods and products. Others found the study of nature through experiment a new and exciting hobby. But the availability of money and time, the growing interest in science and the creation of organizations are not enough to explain the growing success of scientific studies. Historians agree that this rapid growth of science depended upon able men, well-formulated problems and good mathematical and experimental tools.

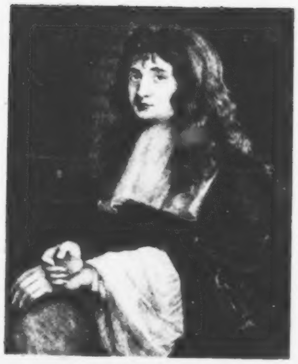
Some of the important scientists who lived between 1600 and 1750 are indicated in the time chart for the Age of Newton. The list includes amateurs and university professors.

Well-formulated problems were numerous in the writings of Galileo and Kepler. Their studies showed how useful mathematics could be when applied to science. They had illustrated the new way of viewing facts and experiments. Furthermore, their works had raised exciting new questions: what forces act on the planets to explain the paths actually observed?, and what assumptions are needed to explain the observed effects of terrestrial gravitation?

Good mathematical and experimental tools were being created. With mathematics being applied to physics, studies in


1600
1650
1700
1750
1642
NEWTON
1727
Historical Events
Settlement of Jamestown
Settlement of Plymouth
Puritan Revolution
Glorious Revolution
Peace of Utrecht
Government
Cardinal Richelieu
Peter the Great of Russia
Oliver Cromwell
Jean Colbert
Louis XIV of France
Science
René Descartes
Edmund Halley
William Harvey
Jean Bernoulli
Johannes Kepler
Blaise Pascal
Robert Boyle
Christiaan Huygens
Galileo
Gottfreid Leibniz
Philosophy
Thomas Hobbes
George Berkeley
Spinoza
Montesquieu
John Locke
David Hume
Voltaire
Literature
Ben Jonson
Jonathan Swift
John Milton
Alexander Pope
Molière
Henry Fielding
John Dryden
Daniel Defoe
Art
Peter Paul Rubens
Jean Watteau
Velazquez
William Hogarth
Bernini
François Boucher
Rembrandt Van Rijn
Christopher Wren
Music
Jean Lully
Henry Purcell
Antonio Vivaldi
Johann Sebastian Bach
George Frederick Handel





Newton entered Trinity College, Cambridge University, in 1661 at the age of eighteen. He soon was doing experiments and teaching while still a student. This early engraving shows the quiet student wearing a wig and heavy academic robes. Trinity students even today wear the same type of robes to their classes. The drawing of the telescope and its parts was done by Newton, while he was still a student.

each field stimulated developments in the other. Similarly, the instrument-maker and the scientist aided each other.

Another factor of great importance was the rapid accumulation of scientific knowledge. Each study could build on those done previously. Thus from the time of Galileo repeatable experiments woven into testable theories were available for study, modification and application.

Newton, who lived in this new scientific age, is the central person in this chapter. However, before we follow Newton's work, we must recall that in science as in any other field, many men made useful contributions. The whole structure of science depends not only upon those whom we call geniuses, but also upon many lesser-known men. As Lord Rutherford, one of the founders of modern atomic theory, said:

> It is not in the nature of things for any one man to make a sudden violent discovery; science goes step by step, and every man depends upon the work of his predecessors....Scientists are not dependent on the ideas of a single man, but on the combined wisdom of thousands of men.

Properly we should trace in each man's contribution his dependence upon those who worked before him, the influence of his contemporaries and his influence upon his successors. While this would be interesting and rewarding, here we can only briefly hint at these relationships.

Fig. 8.1 Chronological table of Newton's era

8.1 A sketch of Newton's life. Isaac Newton was born on Christmas day, 1642, in the small English village of Woolsthorpe in Lincolnshire. He was a quiet farm boy, who, like young Galileo, loved to build and tinker with mechanical gadgets and seemed to have a liking for mathematics. With financial help from an uncle he went to Trinity College of Cambridge University in 1661. There he initially enrolled in the study of mathematics as applied to astrology and he was an eager and excellent student. In 1665, to escape the Black Plague (bubonic plague) which swept through England, Newton went home to the quiet farm in Woolsthorpe. There, by the time he was twenty-four, he had quietly made spectacular discoveries in mathematics (binomial theorem, differential calculus), optics (theory of colors) and mechanics. Referring to this period, Newton once wrote:

See "Newton and the Principia" in Project Physics Reader 2.

> I began to think of gravity extending to the orb of the moon, and...from Kepler's rule [third law]...I deduced that the forces which keep the Planets in their orbs must be reciprocally as the squares of their distances from the centers about which they revolve: and thereby compared the force requisite to keep the moon in her orb with the force of gravity at the surface of the earth, and found them to answer pretty nearly. All this was in the two plague years of 1665 and 1666, for in those days I was in the prime of my age for invention, and minded mathematics and philosophy more than at any time since.

Thus during his isolation the brilliant young Newton had developed a clear idea of the first two Laws of Motion and of the formula for gravitational attraction. However, he did not announce the latter until many years after Huygens' equivalent statement.

This must have been the time of the famous and disputed fall of the apple. One of the better authorities for this story is a biography of Newton written in 1752 by his friend William Stukeley, where we can read that on one occasion Stukeley was having tea with Newton in a garden under some apple trees, when Newton recalled that

In Unit 1 a newton was defined as the force needed to give an acceleration of 1 meter per sec^2 to a 1-kilogram mass. In newtons, what approximately is the earth's gravitational attraction on an apple?

> he was just in the same situation, as when formerly, the notion of gravitation came into his mind. It was occasion'd by the fall of an apple, as he sat in a contemplative mood. Why should that apple always descend perpendicularly to the ground, thought he to himself. Why should it not go sideways or upwards, but constantly to the earth's centre?

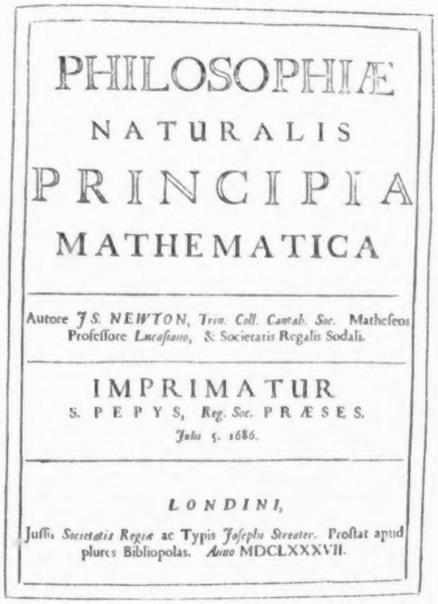
The main emphasis in this story should probably be placed on the word contemplative. Moreover, it fits again the pattern we have seen before: a great puzzle (here, that of the forces acting on planets) begins to be solved when a clear-thinking person contemplates a long-known phenomenon. Where others had seen no relationship, Newton did. Similarly

Galileo used the descent of rolling bodies to show the usefulness of mathematics in science. Likewise, Kepler used a small difference between theory and observation in the motion of Mars as the starting point for a new approach to planetary astronomy.

Soon after Newton's return to Cambridge, he was chosen to follow his teacher as professor of mathematics. He taught at the university and contributed papers to the Royal Society, at first particularly on optics. His Theory of Light and Colors, when finally published in 1672, involved him in so long and bitter a controversy with rivals that the shy and introspective man resolved never to publish anything more (but he did).

Newton made the first reflecting telescope.

In 1684 Newton's devoted friend Halley came to ask his advice in a dispute with Wren and Hooke about the force that would have to act on a body moving along an ellipse in accord with Kepler's laws. Halley was pleasantly surprised to learn that Newton had already derived the rigorous solution to this problem ("and much other matter"). Halley then persuaded his reluctant friend to publish his studies, which solved one of the most debated and interesting scientific questions of the time. To encourage Newton, Halley became responsible for all the costs of publication. In less than two years of incredible labors, Newton had the Principia (Fig. 8.2) ready for the printer. Publication of the Principia in 1687 quickly established Newton as one of the greatest thinkers in history.



PHILOSOPHIÆ
NATURALIS
PRINCIPIA
MATHEMATICA

Autore *J. S. NEWTON*, *Trin. Coll. Cantab. Soc.* Matheſeos
Profeſſore *Lucaſiano*, & Societatis Regalis Sodali.

IMPRIMATUR
S. PEPYS, *Reg. Soc.* PRÆSES.
Julii 5. 1686.

LONDINI,
Juſſu *Societatis Regiæ* ac Typis *Joſephi Streater*. Proſtat apud
plures Bibliopolas. *Anno* MDCLXXXVII.

Fig. 8.2 Title page of Principia Mathematica. Because the Royal Society sponsored the book, the title page includes the name of the Society's president, Samuel Pepys, whose diary is famous.

A few years afterwards, Newton had a nervous breakdown. He recovered, but from then until his death 35 years later, in 1727, he made no major new scientific discoveries. He rounded out earlier studies on heat and optics, and turned more and more to writing on theological chronology. During those years he received honors in abundance. In 1699 he was appointed Master of the Mint, partly because of his great interest in and knowledge about the chemistry of metals, and he helped to re-establish the British currency, which had become debased. In 1689 and 1701 he represented his university in Parliament, and he was knighted in 1705 by Queen Anne. He was president of the Royal Society from 1703 to his death in 1727, and he was buried in Westminster Abbey.

Q1 Why might we conclude that Newton's isolation on the farm during the Plague Years (1665-66) contributed to his scientific achievements?

Q2 What was the important role played by the scientific societies? Do such societies today perform the same functions?

Q3 What was important about Newton's mood when he noticed the apple fall?

8.2 Newton's Principia. In the original preface to Newton's Principia—one of the most important books in the history of science—we find a clear outline of the book:

> Since the ancients (as we are told by Pappus) esteemed the science of mechanics of greatest importance in the investigation of natural things, and the moderns, rejecting substantial forms and occult qualities, have endeavored to subject the phenomena of nature to the laws of mathematics, I have in this treatise cultivated mathematics as far as it relates to philosophy [we would say 'physical science']...for the whole burden of philosophy seems to consist in this—from the phenomena of motions to investigate [induce] the forces of nature, and then from these forces to demonstrate [deduce] the other phenomena, and to this end the general propositions in the first and second Books are directed. In the third Book I give an example of this in the explication of the system of the World; for by the propositions mathematically demonstrated in the former Books, in the third I derive from the celestial phenomena the forces of gravity with which bodies tend to the sun and the several planets. Then from these forces, by other propositions which are also mathematical, I deduce the motions of the planets, the comets, the moon, and the sea [tides]....

See Newton's Laws of Motion and Proposition One in Project Physics Reader 2.

The work begins with a set of definitions—mass, momentum, inertia, force. Next come the three Laws of Motion and the principles of composition of vectors (forces and velocities), which were discussed in Unit 1. Newton then included an equally remarkable and important passage on "Rules of Reasoning in Philosophy." The four rules or assumptions, reflecting his profound faith in the uniformity of all nature, were intended to guide scientists in making hypotheses. These are still useful. The first has been called a Principle of Parsimony; the second and third, Principles of Unity. The fourth is a faith without which we could not use the process of logic.

In a short form, with some modern words, these rules are:

These Rules are stated by Newton at the beginning of Book III of the Principia, p. 398 of the Cajori edition, University of California Press.

I. Nature is essentially simple; therefore we should not introduce more hypotheses than are sufficient and necessary for the explanation of observed facts. "Nature does nothing ...in vain, and more is in vain when less will serve." This fundamental faith of all scientists is almost a paraphrase of Galileo's "Nature...does not that by many things, which may be done by few"—and he, in turn, quoted the same opinion from Aristotle. Thus the rule has a long history.

II. "Therefore to the same natural effects we must, as far as possible, assign the same causes. As to respiration in a man and in a beast; the descent of stones in Europe and in America; ...the reflection of light in the earth, and in the planets."

III. Properties common to all those bodies within reach of our experiments are to be assumed (even if only tentatively) to apply to all bodies in general. Since all physical objects known to experimenters had always been found to have mass, this rule would guide Newton to propose that every object has mass.

IV. In "experimental philosophy," those hypotheses or generalizations which are based on experience are to be accepted as "accurately or very nearly true, notwithstanding any contrary hypotheses that may be imagined" until we have additional evidence by which our hypotheses may be made more accurate, or revised.

The Principia was an extraordinary document. Its three main sections contained a wealth of mathematical and physical discoveries. But overshadowing everything else in the book is the theory of universal gravitation, with Newton's proofs and arguments leading to it. Newton uses a form of argument patterned after that of Euclid—the type of proofs you encountered in your geometry studies. Because the detailed mathematical steps used in the Principia are no longer familiar, the steps which are given below have often been restated in modern terms.

The central idea of universal gravitation can be very simply stated: every object in the universe attracts every other object. Moreover, the magnitude of these attractions depends in a simple way on the distance between the objects. Such a sweeping assertion certainly defies full and detailed verification—for after all, we cannot undertake to measure the forces experienced by all the objects in the universe!

Q4 Write a brief simple restatement of each of Newton's Rules of Reasoning.

Q5 Why is the Principia difficult for us to read?

8.3 A preview of Newton's analysis. We shall now preview Newton's development of his theory of universal gravitation. Also we shall see how he was able to use the theory to unify the main strands of physical science which had been developing independently. As we proceed, notice the extent to which Newton relied on the laws found by Kepler; be alert to the appearance of new hypotheses and assumptions; and watch for the interaction of experimental observations and theoretical deductions. In short, in this notable and yet typical case, aim for an understanding of the process of the construction and verification of a theory; do not be satisfied with a mere memorization of the individual steps.

What were the known laws about motion that Newton unified? He had his own three laws, which you met in Unit 1. Also he had the three Laws of Planetary Motion stated by Kepler, which we considered in Chapter 7.

Newton (from Chapter 3)	Kepler (from Chapter 7)
1. A body continues in a state of rest, or of uniform motion in a straight line, unless acted upon by a net force. (Law of Inertia.)	1. The planets move in orbits which are ellipses and have the sun at one focus.
2. The net force acting on an object is directly proportional to and in the same direction as the acceleration.	2. The line from the sun to a planet sweeps over areas which are proportional to the time intervals.
3. To every action there is an equal and opposite reaction.	3. The squares of the periods of the planets are proportional to the cubes of their mean distances from the sun. $T^2 = k \cdot a^3$ Eq. (7.3).

Now let us preview the argument by which these two sets of laws can be combined.

The curvature of planetary motions requires a net force.

According to Kepler's first law, the planets move in orbits which are ellipses, that is, curved orbits. But according to Newton's first law a change of motion, either in direction or amount, results when a net force is acting. Therefore we can conclude that a net force is continually accelerating all the planets. However, this result does not specify what type of force is acting or whether it arises from some particular center.

Motion influenced by any central force will satisfy the Law of Areas.

Newton's second law combined with Kepler's first two laws was the basis on which Newton achieved a brilliant solution to a very difficult problem. Newton's second law says that in every case the net force is exerted in the direction of the observed acceleration. But what was the direction of the acceleration? Was it toward one center or perhaps toward many? By a geometrical analysis Newton found that any moving body acted upon by a central force, that is, any force centered on a point, when viewed from that point, will move according to Kepler's Law of Areas. Because the areas found by Kepler were measured around the sun, Newton could conclude that the sun at the focus of the ellipse was the source of the central force.

The net force accelerates the planets toward the sun.

Newton considered a variety of force laws centered on points at various places. Some of his results are quite unexpected. For example, he found that circular motion would result from an inverse-fifth power force law, $F \propto 1/R^5$, acting from a point on the circle! But, as we have just seen, the center of the force on the planets had to be at the sun, located at one focus of the elliptical orbits. For an elliptical orbit, or actually for an orbit along any of the conic sections discussed in Chapter 7, the central force from the focus had to be an inverse-square force, $F \propto 1/R^2$. In this way Newton found that only an inverse-square force centered on the sun would result in the observed motions of the planets as described by Kepler's first two laws. Newton then clinched the argument by finding that such a force law would also result in Kepler's third law, the Law of Periods, $T^2 = ka^3$.

If the strength of the force varies inversely with the square of the distance, that is,

$$F \propto 1/R^2,$$

the orbits will be ellipses, or some other conic section.

From this analysis Newton concluded that one general Law of Universal Gravitation, that applied to the earth and apples, also applied to the sun and the planets, and all other bodies, such as comets, moving in the solar system. This was Newton's great synthesis. He brought together the terrestrial laws of motion, found by Galileo and others, and the astronomical laws found by Kepler. One new set of laws explained both. Heaven and earth were united in one grand system dominated by the Law of Universal Gravitation. No wonder that the English poet Alexander Pope wrote:

> Nature and Nature's laws lay hid in night:
> God said, Let Newton be! and all was light.

As you will find by inspection, the Principia was filled with long geometrical arguments and was difficult to read. Happily, gifted popularizers wrote summaries, through which many people learned Newton's arguments and conclusions. In Europe one of the most widely read of these popular books was published in 1736 by the French philosopher and reformer Voltaire.

Readers of these books were excited, and perhaps puzzled by the new approach and assumptions. From ancient Greece until well after Copernicus, the ideas of natural place and natural motion had been used to explain the general position and movements of the planets. The Greeks believed that the planets were in their orbits because that was their proper place. Furthermore, their natural motions were, as you have seen, assumed to be at uniform rates in perfect circles, or in combinations of circles. However, to Newton the natural motion of a body was at a uniform rate along a straight line. Motion in a curve was evidence that a net force was continually accelerating the planets away from their natural motion along straight lines. What a reversal of the assumptions about the type of motion which was "natural"!

This sketch of Newton's argument outlines his procedure and the interweaving of earthly physics with astronomical conclusions from Kepler. Now we shall examine the details of Newton's analysis and begin as he did with a study of the motion of bodies accelerated by a central force.

Q6 Complete the following summary of Newton's analysis:

Step 1. Kepler's ellipses + Newton's first law (inertia) → ?

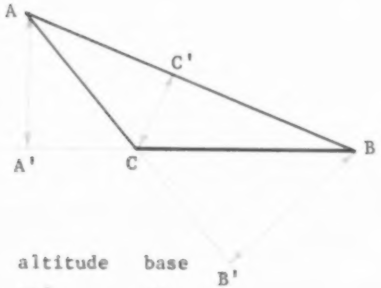
Step 2. Newton's second law (force) + Kepler's area law → ?

Step 3. Add Kepler's ellipses → ?

Step 4. What observed properties of the planetary orbits required that $F \propto 1/R^2$?

Step 5. Does the result agree with Kepler's Law of Periods?

8.4 Motion under a central force. How will a moving body respond to a central force? Before we proceed with the analysis, we need to review one basic property of triangles. The area of a triangle equals ½ base × altitude. But, as Fig. 8.3 shows, any of the three sides can be chosen as the base with the corresponding vertex being the intersection of the other two sides.



altitude	base
CC'	AB
BB'	AC
AA'	BC

Fig. 8.3 The altitude of a triangle is the perpendicular distance from the vertex of two sides to the third side, which is then the base.

1. Suppose that a body, initially at rest at a point Q, is exposed to a brief force, like a hammer blow, directed toward O. The body will be accelerated toward O. It will begin to move toward O, and after some definite time interval Δt, it will have moved a definite distance to a new point Q', i.e., through the distance QQ'. (Fig. 8.4a.)

Fig. 8.4(a)

2. Suppose that the body was initially at some point P and moving at a uniform speed v along the straight line through PQ. (See Fig. 8.4b.) In equal intervals of time, Δt, it will move equal distances, PQ, QR, RS, etc. How will its motion appear to an observer at some point O?

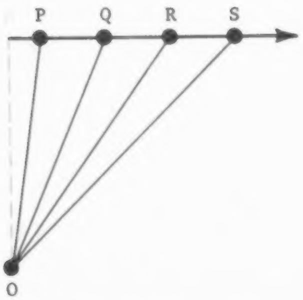


Fig. 8.4(b) A body moving at a uniform rate in a straight line is viewed from point O.

To prepare for what follows—and to make incidentally a discovery that may be surprising—consider first the triangles OPQ and OQR. These have equal bases, PQ and QR; and also equal altitudes, ON. Therefore they have equal areas. And therefore, the line from any observer not on the line, say at point O, to a body moving at a uniform speed in a straight line, like PQR will sweep over equal areas in equal times. Strange as it may seem, Kepler's Law of Areas applies even to a body on which there is no net force, and which therefore is moving uniformly along a straight line.

3. How will the motion of the object be changed if at point Q it is exposed to a brief force, such as a blow, directed toward point O? A combination of the constructions in 1 and 2 above can be used to determine the new velocity vector. (See Fig. 8.4c.) As in 1 above, the force applied

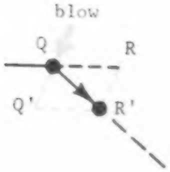


Fig. 8.4(c) A force applied briefly to a body moving in a straight line QR changes the motion to QR'.

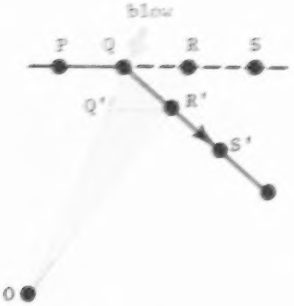


Fig. 8.4(d) The areas of OPQ and OQR' are equal.

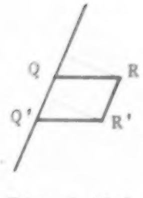


Fig. 8.4(e) The distances of R and R' from QQ' are equal.

at point Q accelerates the object toward the center. In the time interval Δt, a stationary object at Q would move to point Q′. But the object was moving and, without this acceleration, would have moved to point R. Then, as Fig. 8.4c indicates, the resultant motion is to point R′. Fig. 8.4d shows Fig. 8.4c combined with Fig. 8.4b.

Earlier we found that the areas of the triangles OPQ and OQR were equal. Are the areas of the triangles OQR and OQR′ also equal? To examine this question we can consider the perpendicular distances of R and R′ from the line Q′Q as the altitudes of the two triangles. As Fig. 8.4e shows, R and R′ are equally distant from the line Q′Q. Both triangles also have a common base, OQ. Therefore, the areas of triangles OQR and OQR′ are equal. Thus we may conclude that the line from the center of force, point O, to the body sweeps over equal areas in equal time intervals.

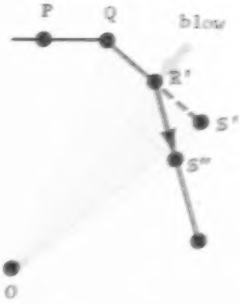


Fig. 8.4(f) A force applied at R' causes the body to move to S''.

If another blow directed toward O, even a blow of a different magnitude, were given at point R′, the body would move to some point S′′, as indicated in Fig. 8.4f. By a similar analysis you can find that the areas of triangles OR′S′′ and OR′S′ are equal. Their areas also equal the area of triangle OPQ.

SG 8.1

In the geometrical argument above we have considered the force to be applied at intervals Δt. What motion will result if each time interval Δt is made vanishingly small and the force is applied continuously? As you would suspect, and as can be shown by rigorous proof, the argument holds for a continuously acting central force. We then have an important conclusion: if a body is continuously acted upon by any central force, it will move in accordance with Kepler's Law of Areas. In terms of the planetary orbits from which the Law of Areas was found, the accelerating force must be a central force. Furthermore, the sun is at the center of the force. Notice that the way in which the strength of the central force depends on distance has not been specified.

Does this conclusion apply if the central force is one of repulsion rather than attraction?

Q7 What types of motion satisfy Kepler's Law of Areas?

8.5 *The inverse-square law of planetary force.* In the last section we found that the force influencing the motion of a planet had to be a central force toward the sun. But many questions remained to be considered. How did the strength of the force change with the planet's distance from the center of motion? Would one general force law account for the motions of all the planets? Or was a different law needed for each planet? Would the force law be consistent with Kepler's third law, $T^2 \propto R^3$? How could this be tested? And from what observations could the amount of the force be determined?

Clearly, the nature of the force law should now be found; but how? The Law of Areas will not be useful, for we have seen that it is satisfied by any central force. But Kepler's first law (elliptical orbits) and third law (the relation between distance and period) remain to be used. We could start by considering what the relation between force and distance must be in order to satisfy the third law. Newton preferred to assume a force law and tested it against Kepler's third law.

See "Newton's Laws of Dynamics" in Project Physics Reader 1.

Some clues suggest what force law to consider first. Because the orbits of the planets are curved toward the center of motion at the sun, the force must be one of attraction toward the sun. While the force might become greater with distance from the sun, we could expect the force to decrease with distance. Possibly the force varies inversely with the distance: $F \propto 1/R$. Or perhaps, like the brightness of a light, the force decreases with the inverse square of the distance: $F \propto 1/R^2$. Possibly the force weakens very rapidly with distance: $F \propto 1/R^3$, or even $F \propto 1/R^n$, where n is some large number. One could try any of these possibilities. But here let us follow Newton's lead and test whether $F \propto 1/R^2$ agrees with the astronomical results.

Newton assumed that:

$$F_{grav} \propto \frac{1}{R^2}.$$

Actually Newton's choice of this inverse-square law was not accidental. Others such as Wren, Halley and Hooke were attempting to solve the same problem and were considering the same force law. In fact, Halley came to Newton in 1684 specifically to ask if he could supply a proof of the correctness of the inverse-square law which the others were seeking in vain. Even if Newton had not already derived the proof, it is likely that someone would have soon. At any one time in the development of science, a rather narrow range of interesting and important problems holds the attention of many scientists in a given field, and often several solutions are proposed at about the same time.

How can you explain the historical fact that many scientific discoveries have been made independently and almost simultaneously by two or more men? What examples can you list?

Because Kepler's third, or Harmonic, law relates the periods of different planets to their distances from the sun, this law should be useful in our study of how the gravitational attraction of the sun changes with distance. We have another clue from Galileo's conclusion that the distance d through which a body moves as a result of the earth's gravitational attraction increases with the square of the time, t^2. The full relation is

$$d = \tfrac{1}{2}at^2,$$

where a is a constant acceleration.

We wish to compare the central forces, or accelerations, acting on two planets at different distances from the sun. Fig. 8.5 indicates the geometry for two planets moving in circular orbits. For convenience we can consider one of the planets to be the earth, E. The other planet P can be at any distance from the sun. According to Newton's Law of Inertia, any planet continually tends to move in a straight line. But we observe that it actually moves in an orbit which is (nearly) a circle. Then, as Fig. 8.5 indicates, in a small interval of time t the planet moves forward, and also falls a distance d toward the sun. No matter how large the orbit is, the motion of each planet is similar to that of the others, though each has its own period T.

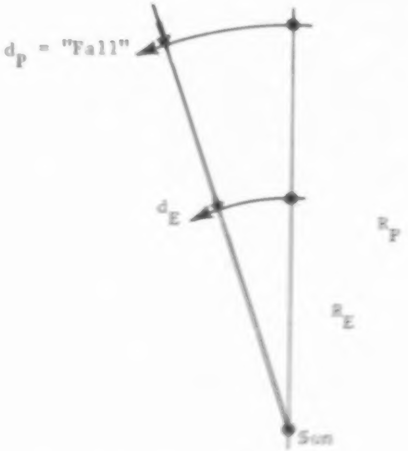


Fig. 8.5 The "fall" of the earth and a planet toward the sun.

This time t taken for either planet to move through the portion of the orbit indicated in Fig. 8.5 is a fraction of the total time T required for the planet to make one revolution around the sun. No matter how large the orbit, the fraction $\frac{t}{T}$ required to move through this angle, will be the same for any planet. But while the planet moves through this arc, it also falls toward the sun through a distance d. From Fig. 8.5 we can also see that the distance d increases in proportion to the distance R.

Now let us use Galileo's equation to compare the "falls" of two planets toward the sun:

$$\frac{d_P}{d_E} = \frac{\frac{1}{2}a_P\ t_P^2}{\frac{1}{2}a_E\ t_E^2}, \text{ or } \frac{d_P}{d_E} = \frac{a_P\ t_P^2}{a_E\ t_E^2}$$

But because the values of d are proportional to R, we can replace the ratio (d_P/d_E) by (R_P/R_E) and have:

$$\frac{R_P}{R_E} = \frac{a_P\ t_P^2}{a_E\ t_E^2}$$

Now, if we can express the a's in terms of R's or t's, we can see whether the result agrees with Kepler's third law.

Assume that $a = 1/R^2$.

Like Newton, let us assume that a varies with $1/R^2$. By rearranging the equation we have:

$$\frac{R_P}{R_E} = \frac{\frac{1}{R_P^2}\ t_P^2}{\frac{1}{R_E^2}\ t_E^2}, \text{ or } \quad \frac{R_P^3}{R_E^3} = \frac{t_P^2}{t_E^2} .$$

This has the same form as Kepler's third law, but involves the times t for the planet to move through a small arc, rather than the full period T. However, we have seen that the times t required for planets to move througn the same arcs of circular orbits were the same fractions $\frac{t}{T}$ of tneir total period. Thus we can replace t by T and have

SG 8.5

$$\frac{R_P^3}{R_E^3} = \frac{T_P^2}{T_E^2}, \text{ or } \quad R_P^3 = k\ T_P^2 ,$$

where the constant k adjusts the equality for the units in which R and T are expressed. This last equation is Kepler's Law of Periods which we saw as Eq. (7.3).

In this derivation we have made a number of assumptions: 1) that Galileo's law of acceleration relating times and distances of falling bodies on the earth applies to the acceleration of the planets toward the sun, and 2) that the acceleration toward the sun changes as $1/R^2$. As a result we have found that the planets should move according to Kepler's Law of Areas. Since they do, we can (by Newton's fourth rule) accept our assumption that the sun's gravitational attraction does change with $1/R^2$.

In the analysis above we substituted $1/R^2$ for a in the general relation $R = at^2$. What results do you get if you set $a = 1/R^n$, where n is any number? What value must n have to satisfy Kepler's Third Law?

We assumed that the orbits were circles. However, Newton showed that any object moving in an orbit that is a conic section (circle, ellipse, parabola or hyperbola) around a center of force is being acted upon by a net force which varies inversely with the square of the distance from the center of force.

Q8 Why is Kepler's third law ($R^2 \propto T^3$) useful for testing how the gravitational acceleration changed with distance from the sun?

Q9 What simplifying assumptions were made in the derivation here?

8.6 Law of Universal Gravitation. Evidently a central force is holding the planets in their orbits. Furthermore, the strength of this central force changes inversely with the square of the distance from the sun. This strongly suggests that the sun is the source of the force—but it does not necessarily require this conclusion. Newton's results might be fine geometry, but so far they include no physical mechanism. The French philosopher Descartes (1596-1650) had proposed an alternate theory that all space was filled with a subtle, invisible fluid which carried the planets around the sun in a huge whirlpool-like motion. This was an extremely attractive idea, and at the time was widely accepted. However, Newton was able to prove that this mechanism could not account for the quantitative observation of planetary motion summarized in Kepler's laws.

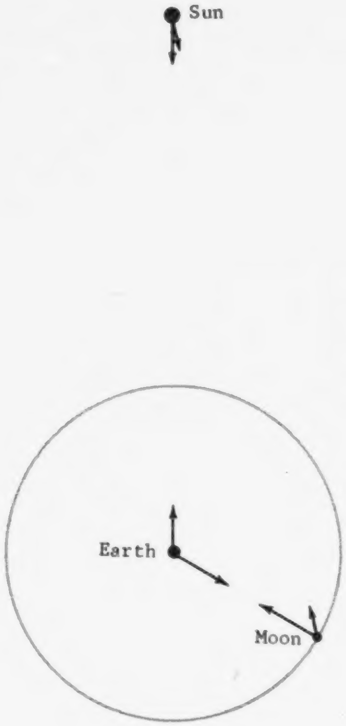
Kepler, you recall, had still a different suggestion. He proposed that some magnetic force reached out from the sun to keep the planets moving. To Kepler this continual push was necessary because he had not realized the nature of inertial motion. His model was inadequate, but at least he was the first to regard the sun as the controlling mechanical agent behind planetary motion. And so the problem remained: was the sun actually the source of the force? If so, on what characteristics of the sun did the amount of the force depend?

At this point Newton proposed a dramatic solution: the force influencing the planets in their orbits is nothing other than a gravitational attraction which the sun exerts on the planets. This is a gravitational pull of exactly the same sort as the pull of the earth on an apple. This assertion, known as the Law of Universal Gravitation, says:

> *every object in the universe attracts every other object with a gravitational force*.

If this is so, there must be gravitational forces between a rock and the earth, between the earth and the moon, between Jupiter and its satellites—and between the sun and each of the planets.

But Newton did not stop by saying only that there is a gravitational force between the planets and the sun. He further claimed that the force is just exactly the right size to account *completely* for the motion of every planet. No other mechanism is needed...no whirlpools in invisible fluids, no magnetic forces. Gravitation, and gravitation alone, underlies the dynamics of the heavens.



The sun, moon and earth each pull on the other. The forces are in matched pairs in agreement with Newton's third law.

Because this concept is so commonplace to us, we are in danger of passing it by without really understanding what it was that Newton was claiming. First, he proposed a truly universal physical law. He excluded no object in the universe from the effects of gravity. Less than a century before it would have been impious or foolish even to suggest that terrestrial laws and forces were the same as those that regulated the whole universe. But Kepler and Galileo had begun the unification of the physics of heaven and earth. Newton was able to carry a step further what had been well begun. Because Newton was eventually able to unite the mechanics of terrestrial objects and the motion of celestial bodies through one set of propositions, his result is called the Newtonian synthesis.

A second feature of Newton's claim, that the orbit of a planet is determined by the gravitational attraction between it and the sun, was to move physics away from geometrical explanations toward physical ones. He shifted the question from "what are the motions?", which Kepler had answered, to "what force effects the motion?" In both the Ptolemaic and Copernican systems the planets moved about points in space rather than about objects, and they moved as they did because they had to by their nature or geometrical shape, not because forces acted on them. Newton, on the other hand, spoke not of points, but of things, of objects, of physical bodies. Without the gravitational attraction of the sun to deflect them continually from straight-line paths, the planets would fly out into the darkness of space. Thus, it was the physical sun which was important rather than the point at which the sun happened to be located.

Newton postulated a specific force. By calling it a force of gravity he was not, however, explaining why it should exist. He seems to be saying essentially this: hold a stone above the surface of the earth and release it. It will accelerate to the ground. Our laws of motion tell us that there must be a force acting on the stone driving it toward the earth. We know the direction of the force and we can find the magnitude of the force by multiplying the mass of the stone by the acceleration. We can give it a name: weight, or gravitational attraction to the earth. Yet the existence of this force is the result of some unexplained interaction between the stone and the earth. Newton assumed, on the basis of his Rules of Reasoning, that the same kind of force exists between the earth and moon, or any planet and the sun. The force drops off as the square of the distance, and is of just the right amount to explain the motion of the planet. But why such a force should exist remained a puzzle, and is still a puzzle today.

Newton's claim that there is a mutual force between a planet and the sun raised a new question. How can a planet and the sun act upon each other at enormous distances without any visible connections between them? On earth you can exert a force on an object by pushing it or pulling it. We are not troubled when we see a cloud or a balloon drifting across the sky even though nothing seems to be touching it. Although air is invisible, we know that it is actually a material substance, which we can feel when it blows against us. Objects falling to the earth and iron objects being attracted to a magnet are more troublesome examples. Still, the distances are usually short, and one could imagine that there is some sort of tenuous contact or influence involved. But the earth is over 90 million miles and Saturn more than 2 billion miles from the sun. How could there possibly be any physical contact between such distant objects? How can we account for such "action at a distance"?

How can forces act at a great distance through space that seems to be empty?

There were in Newton's time, and for a long time afterward, a series of suggestions to explain how mechanical forces could be exerted at such distances. Newton himself, however, did not claim to have discovered how the gravitational force he had postulated was transmitted through space. At least in public, Newton refused to speculate on how the postulated gravitational force was transmitted through space. He saw no way to reach any testable answer which would replace the unacceptable whirlpools of Descartes. As he said in a famous passage in the General Scholium added to his second edition of the Principia (1713):

> ...Hitherto I have not been able to discover the cause of those properties of gravity from phenomena, and I frame no hypotheses; for whatever is not deduced from the phenomena is to be called an hypothesis; and hypotheses, whether metaphysical or physical, whether of occult qualities or mechanical, have no place in experimental philosophy. In this philosophy particular propositions are inferred from the phenomena, and afterwards rendered general by induction. Thus it was that the impenetrability, the mobility, and the impulsive force of bodies, and the laws of motion and of gravitation were discovered. And to us it is enough that gravity does really exist, and act according to the laws which we have explained, and abundantly serves to account for all the motions of the celestial bodies, and of our sea.

This quotation is part of an important comment, or General Scholium, beginning on page 543 of the Cajori edition, University of California Press. The quotation is on page 547.

We quoted Newton at length because this particular passage is frequently misquoted and misinterpreted. The original Latin reads: hypotheses non fingo. This means: "I frame no hypotheses," "I do not feign hypotheses," or "I do not make false hypotheses."

Newton did make numerous hypotheses in his numerous publications, and his letters to friends contain many other speculations which he did not publish. More light is shed on his purpose in writing the General Scholium by his manuscript first draft (written in January, 1712-13). Here Newton very plainly confessed his inability to unite the hypothesis of gravitational forces with observed phenomena. He wrote:

> I have not yet disclosed the cause of gravity, nor have I undertaken to explain it, since I could not understand it from the phenomena.

For us as for Newton the questions which we can answer are those specifying the quantitative nature of this force. We can test the results by applying them to a few selected cases.

Q10 Why do the quantitative predictions from Newton's Law of Universal Gravitation cause us to accept this universal law?

Q11 Why was the explanation of "action at a distance" of interest to scientists?

Q12 Why didn't Newton explain gravitation?

8.7 The magnitude of planetary force.

We want to build a physics of the heavens which explains the motion of bodies by the forces between them. Therefore the general statement that a universal gravitational force exists has to be turned into a quantitative one that gives an expression for both the magnitude and direction of the force between any two objects. It was not enough to assert that a mutual gravitational attraction exists between the sun and say Jupiter. Newton had to specify what quantitative factors determine the size of that mutual force, and how it could be measured, either directly or indirectly.

While studying Newton's Laws of Motion in Unit 1, you first encountered the concept of mass. Indeed, Newton's second law, $F_{net} = ma$, states that the acceleration of any object depends upon the net force and the mass. Consider the two spheres in Fig. 8.6. Let us say that body B has double the mass of body A. Newton's second law tells us that if a net force causes body A to be accelerated a certain amount, double that force will be needed to accelerate body B by the same amount. If we drop bodies A and B, the earth's gravitational attraction causes them to accelerate equally: $a_A = a_B = g$, so the force on B must be twice the force on A. Twice the mass results in twice the force—which suggests that mass itself is the key factor in determining the magnitude of the gravitational force.

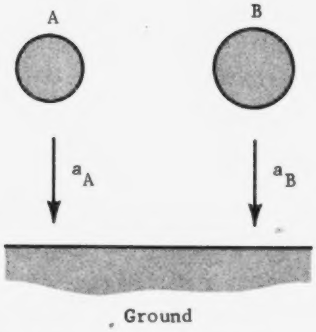


Fig. 8.6. Because falling spheres of unequal mass accelerate at the same rate, the gravitational forces on them must be proportional to their masses.

Like Newton, let us propose that the gravitational force on a planet, due to the pull of the sun, must be proportional to the mass of the planet itself. It immediately follows that this force is also proportional to the mass of the sun. We can see that the second proportionality follows, if we consider the problem in terms of a stone and the earth. We saw before that the downward force of gravity on the stone

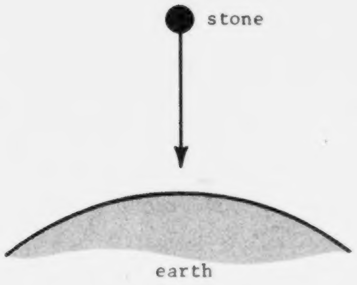
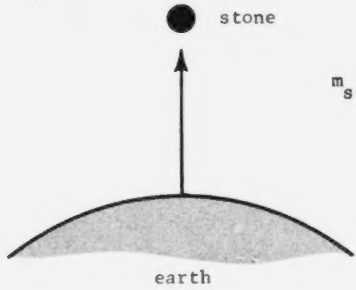


Fig. 8.7(a) The gravitational force on the earth is equal and opposite to the gravitational force on the stone.



$$m_{stone} \times a_{stone} = \text{force}\left\{\begin{matrix}\text{of stone}\\ \text{on earth}\\ \text{or}\\ \text{of earth}\\ \text{on stone}\end{matrix}\right\} = m_{earth} \times a_{earth}$$

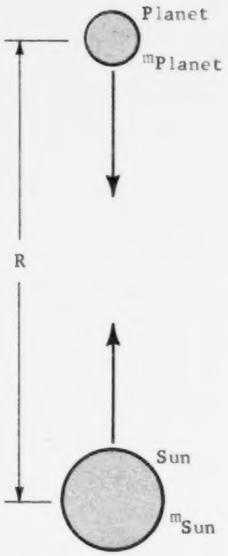


Fig. 8.7(b) The gravitational force on a planet is equal and opposite to the gravitational force, due to the planet, on the sun.

is proportional to the mass of the stone. Then by Newton's third law the force experienced by the earth owing to the stone is equally large and opposite, or upward. Thus while the stone falls, the earth rises. If the stone were fixed in space and the earth free to move, the earth would rise toward the stone until they collided. As Fig. 8.7 indicates, "the forces are equal and opposite," and the accelerations are inversely proportional to the masses.

The conclusion that the forces are equal and opposite, even between a very large mass and a small mass, may seem contrary to common sense. Therefore, let us consider the force between Jupiter and the sun, whose mass is about 1000 times that of Jupiter. As Fig. 8.8 indicates, we could consider the sun as a globe containing a thousand Jupiters. Let us call the force between two Jupiter-sized masses, separated by the distance between Jupiter and the sun, as one unit. Then Jupiter pulls on the sun (a globe of 1000 Jupiters) with a total force of 1000 units. Because each of the 1000 parts of the sun pulls on the planet Jupiter with one unit the total pull of the sun on Jupiter is also 1000 units. Remember that each part of the massive sun not only pulls on the planet, but is also pulled upon by the planet. The more mass there is to attract, the more there is to be attracted.

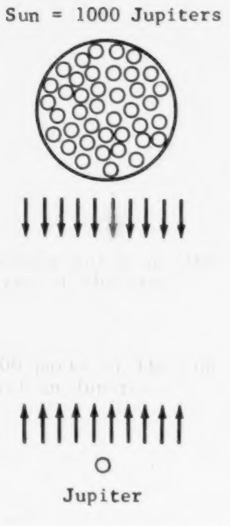


Fig. 8.8 The gravitational forces between the sun and a planet are equal and opposite.

SG 8.3

Force depends on mass of sun.

Force depends on mass of planet.

The gravitational attraction is proportional to the mass of the large body and the attraction is also proportional to the mass of the small body. How do we combine these two proportionalities to get one expression for the total force? Do we multiply the masses, add them, or divide one by the other?

SG 8.7

Figure 8.8 already suggests the answer. If we replaced the sun (1000 Jupiters) by only one Jupiter, the force it would exert on the planet would be only one unit. Likewise, the planet would attract this little sun with a force of only one unit. But because the sun is a thousand times larger than Jupiter, each pull is 1000 units. If we could make the planet Jupiter three times more massive, what would the force be? You probably answered immediately: 3000 units. That is, the force would be multiplied three times. Therefore we conclude that the attraction increases in direct proportion to increases in the mass of either body, and that the total force depends upon the product of the two masses. This conclusion should not be surprising. If you put one brick on a scale, it has a certain weight (a measure of the earth's gravitational attraction on the brick). If you put three bricks on the scale, what will they weigh? That is, how much more will the earth attract the three bricks compared to one brick?

F_{total} varies with $(m_P \times m_S)$. (8.1)

Thus far we have concluded that the force between the sun and a planet will be proportional to the product of the masses [Eq. (8.1)]. Earlier we concluded that this force also depends upon the inverse square of the distance between two bodies. Once again we multiply the two parts to find one force law [Eq. (8.2)] that relates both masses and distance.

$$F_{grav} \propto \frac{m_P \times m_S}{R^2}\,. \qquad (8.2)$$

$$F_{grav} = G\,\frac{m_P \times m_S}{R^2}\,. \qquad (8.3)$$

Such a proportionality as (8.2) can be changed into an equation by introducing a gravitational constant, G, to allow for the units of measurement used. Equation (8.3) is a bold assertion that the force between the sun and any planet depends only upon the masses of the sun and planet and the distance between them. We should notice what the equation omits. The force does not depend upon the name of the planet or its mythological identifications. Furthermore, it does not consider the mass of sun or earth as being in any way special compared with the mass of some other planet.

Try a thought experiment. Consider the possibilities that the force could depend upon the masses in either of two other ways:

(a) total force depends on $(m_{sun} + m_{planet})$; or

(b) total force depends on (m_{sun}/m_{planet}).

Now in imagination let one of the masses become zero. On the basis of these choices, would there still be a force even though there were only one mass left? Could you speak of a gravitational force when there was no body there to be accelerated?

According to Eq. (8.3) the gravitational force is determined by and only by the masses of the bodies and the distance separating them. This equation seems unbelievably simple when we remember the observed complexity of the planetary motions. Yet every one of Kepler's Laws of Planetary Motion is consistent with this relation, and this is the real test whether or not Eq. (8.3) is useful.

Moreover, Newton's proposal that such a simple equation defines the forces between the sun and planets is not the final step. He believed that there was nothing unique or special about the mutual force between sun and planets, or the earth and apples: a relation just like Eq. (8.3) should apply universally to any two bodies having masses m_1 and m_2 separated by a distance R that is large compared to the diameters of the two bodies. In that case we can write a "general law of universal gravitation" [Eq. (8.4)]. The numerical constant G, called the Constant of Universal Gravitation, is assumed to be the same everywhere, whether the objects are two sand grains, two members of a solar system, or two galaxies separated by half a universe. As we shall see, our faith in this simple relationship has become so great that we assume Eq. (8.4) applies everywhere and at all times, past, present and future.

The general Law of Universal Gravitation:

$$F_{grav} = \frac{Gm_1m_2}{R^2} . \tag{8.4}$$

SG 8.2

Even before we gather the evidence supporting Eq. (8.4), the sweeping majesty of Newton's theory of universal gravitation commands our wonder and admiration. You may be curious as to how such a bold universal theory can be tested. The more diverse these tests are, the greater will be our growing belief in the correctness of the theory.

Q13 According to Newton's Law of Action and Opposite Reaction, the earth should rise toward a falling stone. Why don't we observe the earth's motion toward the stone?

Q14 What meaning do you give to G, the Constant of Universal Gravitation?

8.6 Testing a general law. To make a general test of an equation such as Eq. (8.4) we would need to determine the numerical value of all quantities represented by the symbols on both the left and right side of the equality sign. Also we should do this for a wide variety of cases to which the law is supposed to apply and check to see if the values always come out equal on both sides. But we surely cannot proceed that way. How would we determine the magnitude of F_{grav} acting on celestial bodies, except through the application of this equation itself? Newton faced this same problem. Furthermore, he had no reliable numerical values for the masses of the earth and the sun, and none for the value of G.

But worst of all—just what does R represent? As long as we deal with particles or objects so small that their size is negligible compared with the distance between them, there is no ambiguity. But even between the sun and earth, it is not self-evident whether R should refer to the distance from surface to surface, or center to center, or some other distance.

To answer this question, Newton turned to a model—and we can follow his argument in general terms. Let us imagine a rigid homogeneous spherical body, or one composed of spherical shells, such as the body in Fig. 8.9. Within it, any small particle, A, is attracted by all the other particles, near and distant. For each attracting particle B on one side of a line to the center C, there is a symmetrically placed particle D exerting an equal attraction on the other side of the center line. Therefore the net force on particle A is in the direction of the center C.

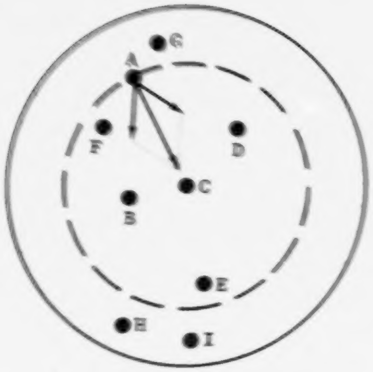


Fig. 8.9 The net gravitational force on a point, A, in a spherical body is towards the center, C.

The amount of force on A is, however, more difficult to determine. Some points, like G, are relatively near to A, while many more, like H and I are farther away. To sum up all the small attractions Newton developed a new mathematical procedure called the infinitesimal calculus. With it he concluded that the the force on a particle at a point A depended only upon the mass of material closer than A to the center. The attractions of the material located further from the center than point A canceled out.

Assume that a planet is a sphere of uniform density (which the earth is not). Compute the gravitational attraction at points 3/4, 1/2, 1/4 of the radius R from the center. The volume of a sphere is given by: Volume = $4\pi R^3/3$.

A similar analysis shows that on an external spherical object—an apple on a branch, or the moon, or a planet—the total gravitational attraction acts as if it originated at the center of the attracting spherical body. Then the distance R in the Law of Universal Gravitation is the distance between centers.

This is a very critical discovery. Now we can consider the gravitational attraction from a rigid spherical body as though its mass was concentrated at a point, called a mass-point.

Q15 What difficulties do we have in testing a universal law?

Q16 What is meant by the term "mass point"?

Q17 How did Newton go about devising some tests of his Theory of Universal Gravitation even though he did not know the value of G? (See next section.)

8.8 The moon and universal gravitation. Before we digressed to discuss what measurements we should use for distance R, we were considering how Newton's equation for universal gravitational attraction could be tested experimentally. To use Eq. (8.4) to find any masses the value of G must be known. However, the value of G was not determined for more than a hundred years after Newton proposed his equation.

Newton had to test Eq. (8.4) indirectly. Since he could not measure any of the attractions directly, he had to be satisfied with comparing various similar motions. This is a procedure often used in scientific studies. Newton began by comparing the gravitational pull of the earth on a terrestrial object (like an apple) and on the moon. From the data available to him, Newton knew that the distance between the center of the earth and the center of the moon was nearly 60 times the radius of the earth. Since the attractive force was believed to vary as $1/R^2$, the gravitational acceleration of the earth upon the moon should be only $1/60^2$ or 1/3600 of that upon an apple at the surface of the earth. From observations of falling bodies it was known that the gravitational acceleration at the earth's surface was nearly 9.80 meters per second per second. Therefore, because of the earth's attraction, in one second the moon should, as Fig. 8.10 indicates, "fall" toward the earth at the rate of 9.80/3600 meters per second^2, or 2.72×10^{-3} m/sec^2. Does it?

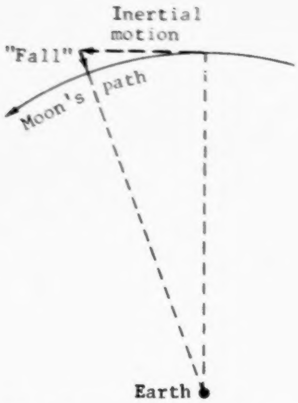


Fig. 8.10 As the moon moves through space, the gravitational attraction of the earth causes the moon to "fall" toward the earth. The continuous combination of straightline inertial motion and "fall" produce the curved orbit.

Predicted value of the earth's gravitational acceleration at the distance of the moon: 2.72×10^{-3} m/sec^2.

As starting information Newton knew that the orbital period of the moon was very nearly $27\frac{1}{3}$ days. Also, he knew that the moon's average distance from the earth is nearly 240,000 miles. But most important was the equation [Eq. (8.5)] for centripetal acceleration, a_c, that you first saw in Unit 1. That is, the acceleration, a_c, toward the center of attraction equals the square of the speed along the orbit, v, divided by the distance between centers, R. To find the average linear speed, v, we divide the total circumference of the moon's orbit, $2\pi R$, by the moon's period, T. Then upon substitution for v^2 in Eq. (8.5) we find Eq. (8.6) for the centripetal acceleration in terms of the radius of the moon's orbit and the orbital period of the moon. When we substitute the known quantities and do the arithmetic we find

$$a_c = v^2/R \qquad (8.5)$$

$$v = 2\pi R/T, \text{ and } v^2 = 4\pi^2 R^2/T^2$$

$$a_c = 4\pi^2 R/T^2 \qquad (8.6)$$

$$a_c = 2.74 \times 10^{-3} \text{ m/sec}^2.$$

Observed rate of moon's acceleration toward the earth: 2.74×10^{-3} m/sec^2.

From Newton's values, which were about as close as these, he concluded that he had

> compared the force requisite to keep the moon in her orbit with the force of gravity at the surface of the earth, and found them to answer pretty nearly.
>
> Therefore the force by which the moon is retained in its orbit becomes, at the very surface of the earth, equal to the force of gravity which we observe in heavy bodies there. And therefore (by Rules [of Reasoning] 1 and 2) the force by which the moon is retained in its orbit is that very same force which we commonly call gravity....

By this one comparison Newton had not yet proved his law of universal gravitational attraction. However, Newton was

SG 8.4 able to show, as we did in Section 8.5, that the general statement of Eq. (8.4) applied also to the motion of planets around the sun along orbits that are conic sections (in this SG 8.6 case, ellipses).

Q18 In what way did Newton compare the motions of a falling apple and of the moon?

Q19 Explain the significance of the numerical results of Newton's computation of the moon's acceleration.

8.10 Gravitation and planetary motion.

The line of discussion used to consider the force of the earth's gravitational attraction upon the moon opens up a further opportunity to check Newton's theory. Newton made a very important assumption: the gravitational attraction must be the cause of centripetal acceleration.

See "Gravity Experiments" in Project Physics Reader 2.

$$F_c = \text{mass} \times \text{acceleration} = m_P a_c = m_P (4\pi^2 R/T^2) \qquad (8.7)$$

$$F_{grav} = G \frac{m_P m_S}{R^2} \qquad (8.3)$$

$$\text{but} \qquad F_c = F_{grav}$$

$$\text{so} \qquad \cancel{m_P} \frac{4\pi^2 R}{T^2} = G \frac{\cancel{m_P} m_S}{R^2}$$

$$T_P^2 = \left[\frac{4\pi^2}{Gm_S}\right] R^3 \qquad (8.8)$$

$T_P^2 = kR^3$ is Kepler's Third Law.

In Chapter 7 we wrote $T^2 = ka^3$, where a was the mean distance in the orbit. However, in Chapter 8 we have used a for acceleration. To lessen confusion we use R here to describe the mean distance of an orbit.

Equation (8.6) related the centripetal acceleration, a_c, to the distance and period of a body moving in a circle. Therefore, for a planet going in a circular orbit—a good approximation for this purpose—the centripetal force F_c is just the acceleration multiplied by the mass of the planet [Eq. (8.7)]. Earlier we found that the gravitational force on a planet in its orbit around the sun was given by Eq. (8.3). Now we can equate the relations, Eqs. (8.3) and (8.7), for F_{grav} and F_c. After canceling the quantity m_P on both sides and rearranging the symbols, we have Eq. (8.8) which has the form of Kepler's third law, $T^2 = kR^3$. The quantity within the bracket of Eq. (8.8) occupies the same place as the constant k which we earlier found to be the same for all planets. The brackets contain only the terms: G, which is supposed to be a universal constant; m_S, the mass of the sun; and the numbers $4\pi^2$. Since none of the terms in the brackets depends upon the particular planet, the bracket gives a value which applies to the sun's effect on any planet.

This does not prove that $k = 4\pi^2/Gm_S$. If, however, we tentatively accept the relation, it follows that $Gm_S = 4\pi^2/k$. Therefore, while we have not seen how one could determine either G or m_S separately, the determination of G would allow us to calculate the mass of the sun, m_S.

$$F_{grav} = G \frac{m_1 m_2}{R^2} \qquad (8.4)$$

We must still question whether G is really a universal constant, i.e., one with the same value for all objects that interact according to Eq. (8.4). Although Newton knew in principle how one might measure the numerical value of G, he lacked the precision equipment necessary. However, he did provide a simple argument in favor of the constancy of G. Consider a body of mass m_1 on the surface of the earth (of mass m_E), at a distance R_E from the earth's center. The body's weight, which we call F_{grav}, is given by m_1g. Then we can again equate the earthly gravitational force, m_1g, with the force predicted by the Law of Universal Gravitation. The result is Eq. (8.9). At any one position on earth, (R_E^2/m_E) is constant (because R_E and m_E are each constant) regardless of what the numerical value of this ratio may be. Then, if all substances at that place show precisely the same value for g, the gravitational acceleration in free fall, the quantity G also must be constant there. This conclusion should hold regardless of the chemical composition, texture, shape and color of the bodies involved in free fall. That g is constant at a given location is just what Newton showed experimentally. His measurements were made not by just dropping small and large bodies, from which Galileo had previously concluded that g cannot vary significantly. Instead Newton used the more accurate method of timing pendulums of equal lengths but of different materials such as wood and gold. After exhaustive experiments, all pointing to the constancy of g, and therefore of G at a given location, Newton could write:

Weight = $F_{grav} = m_1g$, but also

$F_{grav} = G\dfrac{m_1m_E}{R_E^2}$, then

$\cancel{m_1}g = G\dfrac{\cancel{m_1}m_E}{R_E^2}$, and

$$G = \left[\frac{R_E^2}{m_E}\right]g \qquad (8.9)$$

SG 8.8

SG 8.9

> This [constancy] is the quality of all bodies within the reach of our experiments; and therefore (by Rule 3) to be affirmed of all bodies whatsoever.

Thus G attained the status of the Universal Constant of Gravitation—one of the very few universal constants in nature.

SG 8.10

Q20 How did Newton use the centripetal force in his analysis of the motions of the moon and planets?

Q21 On what basis did Newton conclude that G was a universal constant?

Q22 Since the value of g is not the same at all places on the earth, does this mean that perhaps G is not really constant?

8.11 The tides. The flooding and ebbing of the tides, so important to navigators, tradesmen and explorers through the ages, had remained a mystery despite the studies of such men as Galileo. Newton, however, through the application of the Law of Gravitation, was able to explain the main features of the ocean tides. These he found to result from the attraction of the moon and sun upon the fluid waters of the earth. Each day two high tides normally occur. Also, twice each month, when the moon, sun and earth are in line, the tides are higher than the average. Near quarter moon, when the directions from the earth to the moon and sun differ by about 90°, the tidal changes are smaller than average.

What are the phases of the moon when the moon, sun and earth are in line?

Two questions about tidal phenomena demand special attention. First, why do high tides occur on both sides of the earth, including the side away from the moon? Second, why does the time of high tide occur some hours after the moon has crossed the north-south line (meridian)?

Newton realized that the tide-raising force would be the difference between the pull of the moon on the whole solid earth and on the fluid waters at the earth's surface. The moon's distance from the solid earth is 60 earth radii. On the side of the earth nearer the moon, the distance of the water from the moon is only 59 earth radii. On the side of the earth away from the moon the water is 61 earth radii from the moon. Then the accelerations would be those shown in Fig. 8.11. On the side near the moon the acceleration is greater than that on the rigid earth as a whole, so the fluid water on the surface of the earth has a net acceleration toward the moon. On the far side of the earth, the acceleration is less than it is on the earth as a whole, so the water on the far side has a net acceleration away from the moon. We could say that the rigid earth is pulled away from the water.

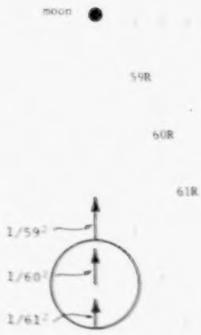


Fig. 8.11 Tidal forces. The earth-moon distance indicated in the figure is greatly reduced because of the space limitations.

If you have been to the seashore or examined tide tables, you know that the high tide does not occur when the moon crosses the north-south line, but some hours later. To explain this even qualitatively we must remember that the oceans are not very deep. As a result, the waters moving in the oceans in response to the moon's attraction encounter friction from the ocean floors, especially in shallow water, and consequently the high tide is delayed. In any particular place the amount of the delay and of the height of the tides depends greatly upon the ease with which the waters can flow. No general theory can be expected to account for all the particular details of the tides. Most of the local predictions in the tide tables are based upon the cyclic variations recorded in the past.

SG 8.16

Since there are tides in the fluid seas, you may wonder if there are tides in the fluid atmosphere and in the earth itself. There are. The earth is not completely rigid, but bends about like steel. The tide in the earth is about a foot high. The atmospheric tides are generally masked by other weather changes. However, at heights near a hundred miles where satellites have been placed, the thin atmosphere rises and falls considerably.

Q23 Why do we consider the acceleration of the moon on the ground below a high tide to be the acceleration of the moon on the earth's center?

Q24 Why is there a high tide on the side of the earth away from the moon?

8.12 Comets. Comets, whose unexpected appearances had through antiquity and the Middle Ages been interpreted as omens of disaster, were shown by Halley and Newton to be nothing more than some sort of cloudy masses that moved around the sun according to Newton's Law of Gravitation. They found that most comets were visible only when closer to the sun than the distance of Jupiter. Several of the very bright comets were found to have orbits that took them well inside the orbit of Mercury to within a few million miles of the sun, as Fig. 8.12 indicates. Some of the orbits have eccentricities near 1.0 and are almost parabolas, and those comets have periods of thousands or even millions of years. Other faint comets have periods of only five to ten years. Unlike the planets, whose orbits are nearly in the plane of the ecliptic, the planes of comet orbits are tilted at all angles to the ecliptic. In fact, about half of the long-period comets move in the direction opposite to the planetary motions.

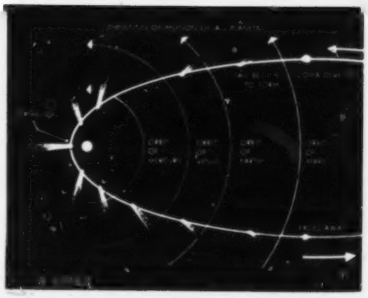
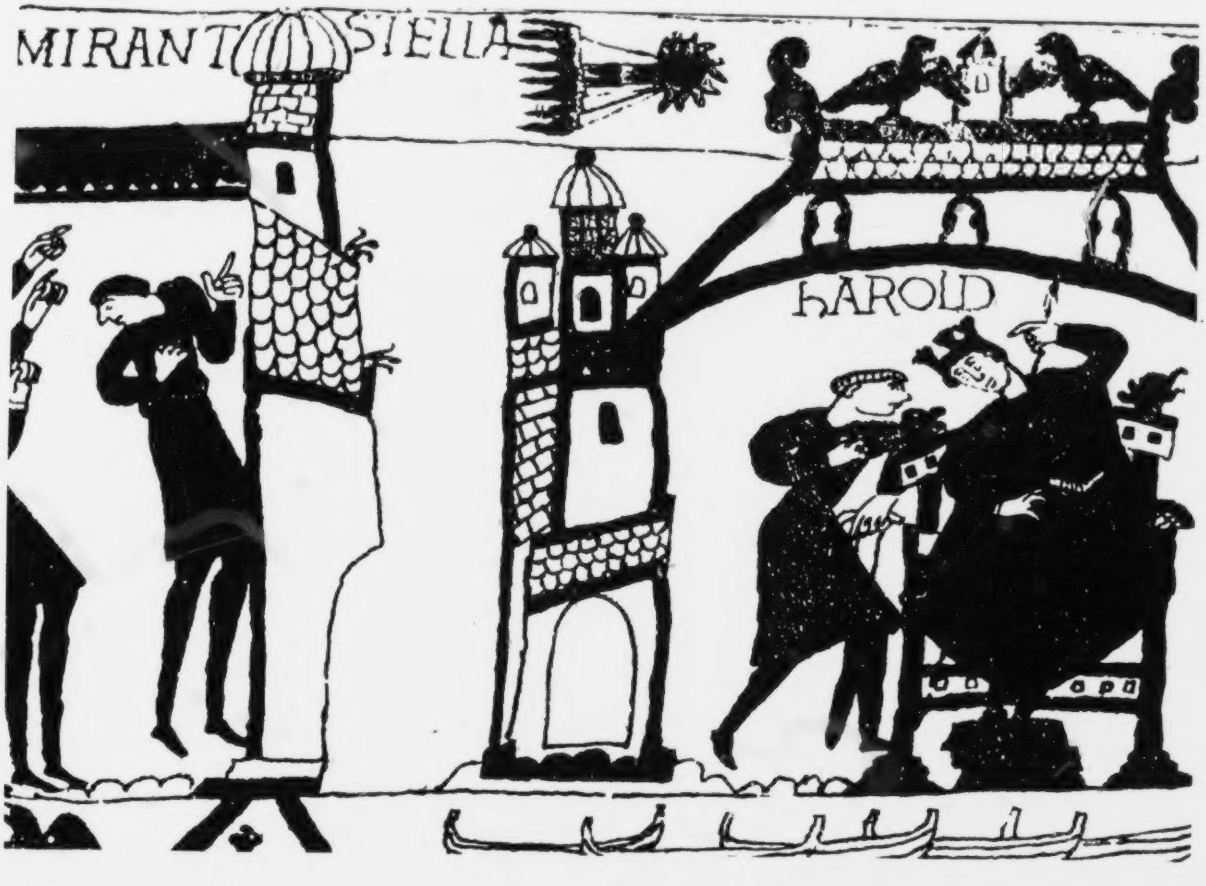


Fig. 8.12 Schematic diagram of the orbit of a comet projected onto the ecliptic plane; comet orbits are tilted at all angles.

Edmund Halley applied Newton's gravitational theory to the motion of bright comets. Among those he studied were the comets of 1531, 1607 and 1682 whose orbits he found to be very nearly the same. Halley suspected that these might be the same comet seen at intervals of about 75 years. If this were the same comet, it should return in about 1757—as

Fig. 8.13 A scene from the Bayeux tapestry, which was embroidered about 1070. The bright comet of 1066 can be seen at the top of the figure. This comet was later identified as being Halley's comet. At the right, Harold, pretender to the throne of England, is warned that the comet is an ill omen. Later that year at the Battle of Hastings Harold was defeated by William the Conqueror.



it did—after moving out to 35 times the earth's distance from the sun. This is Halley's comet, due to be near the sun and bright again in 1985 or 1986.

With the period of this bright comet known, its approximate dates of appearance could be traced back in history. In the records kept by Chinese and Japanese this comet has been identified at every expected appearance except one since 240 B.C. That the records of such a celestial event are incomplete in Europe is a sad commentary upon the interests and culture of Europe during the so-called "Dark Ages." One of the few European records of this comet is the famous Bayeux tapestry, embroidered with 72 scenes of the Norman Conquest of England in 1066, which shows the comet overhead and the populace cowering below (Fig. 8.13). A major triumph of Newtonian science was its use to explain comets, which for centuries had been frightening events.

See "Boy Who Redeemed His Father's Name" in Project Physics Reader 2.

See "Great Comet of 1965" in Project Physics Reader 2.

Q25 In what way did Halley's study of comets support Newton's theory?

8.13 Relative masses of the planets compared to the sun. The masses of the planets having satellites can be compared to the mass of the sun by the use of Eq. (8.8) even though we do not know the value of the Universal Gravitational Constant, G. This we can do by forming ratios involving the periods and distances of the planets around the sun and of satellites around the planets. As a start we rearrange Eq. (8.8) into the form of Eq. (8.10). This relates the period of a planet to its distance from the sun and the total mass of the sun plus planet, but the mass of the planet can be neglected. If we change the subscripts, this relation applies equally well to a planet P and one of its satellites, Sat. Then we may find the ratio of the mass of the sun to the mass of the planet:

$$T^2 = \left[\frac{4\pi^2}{Gm_S}\right] R^3 \qquad (8.8)$$

By rearrangement, this becomes

$$m_S = \left[\frac{4\pi^2}{G}\right]\frac{R^3}{T^2} \quad . \qquad (8.10)$$

$$\frac{\text{mass of the sun-planet pair}}{\text{mass of the planet-satellite pair}} = \frac{m_{sun}}{m_P} = \frac{\left[\frac{4\pi^2}{G}\right]\frac{R_P{}^3}{T_P{}^2}}{\left[\frac{4\pi^2}{G}\right]\frac{R_{Sat}{}^3}{T_{Sat}{}^2}} .$$

The brackets cancel. After some rearrangement we have

$$\frac{m_{sun}}{m_P} = \left[\frac{T_{Sat}{}^2}{T_P{}^2}\right]\left[\frac{R_P{}^3}{R_{Sat}{}^3}\right]. \qquad (8.11)$$

Now, for example, we can determine the relative mass of Jupiter and the sun. We know the period T_S and the distance R_S between Jupiter and one of its satellites—Callisto, one

$$\frac{m_{sun}}{m_P} = \left[\frac{R_P}{R_{Sat}}\right]^3 \left[\frac{T_{Sat}}{T_P}\right]^2 \qquad (8.11)$$

$$\left[\frac{R_P}{R_{Sat}}\right] = \frac{483{,}000{,}000 \text{ mi}}{1{,}170{,}000 \text{ mi}} = 413$$

$$\left[\frac{T_{Sat}}{T_P}\right] = \frac{16.71 \text{ d}}{4{,}332 \text{ d}} = \frac{1}{259}$$

then

$$\frac{m_{sun}}{m_P} = (413)^3 \times \left[\frac{1}{259}\right]^2$$

$$\frac{m_{sun}}{m_P} = \frac{7.05 \times 10^7}{6.70 \times 10^4} = 1050.$$

of the Galilean satellites. Also we know the orbital period T_p and the distance R_p of Jupiter from the sun. Table 8.1 presents the modern data. In the margin we have worked out the arithmetic.

SG 8.11
SG 8.12

Table 8.1. Data on the Motion of Callisto around Jupiter, and on the Motion of Jupiter around the Sun.

Object	Period (T), days	Distance, R, miles
Callisto	16.71	1,170,000
Jupiter	4,332	483,000,000

In this way Newton found the masses of Jupiter, Saturn and the earth compared to the sun's mass to be: 1/1067, 1/3021 and 1/169,282. (The modern values are: 1/1048, 1/3499 and 1/331,950.) Thus the application of gravitational theory permitted for the first time a determination of the relative masses of the sun and planets.

Newton's relative value for the mass of the earth was in error because the distance from the earth to the sun was not accurately known.

Q26 Even though Newton did not know the value of G, how could he use Eq. (8.10),

$$m_S = \left[\frac{4\pi^2}{G}\right] \frac{R^3}{T^2} \quad ,$$

to derive relative masses of some planets compared to the sun? For which planets could he find such masses, in terms of the sun's mass?

Q27 If the period of a satellite of Uranus is 1 day 10 hours and its mean distance from Uranus is 81,000 miles, what is the mass of Uranus compared to the sun's mass?

8.14 The scope of the principle of universal gravitation. Although Newton made numerous additional applications of his Law of Universal Gravitation, we cannot consider them in detail here. He investigated the causes of the irregular motion of the moon and showed that its orbit would be continually changing. As the moon moves around the earth, the moon's distance from the sun changes continually. This changes the net force of the earth and sun on the orbiting moon. Newton also showed that other changes in the moon's motion occur because the earth is not a perfect sphere, but has an equatorial diameter 27 miles greater than the diameter through the poles. Newton commented on the problem of the moon's motion that "the calculation of this motion is difficult." Even so, he obtained predicted values in reasonable agreement with the observed values available at his time.

Newton investigated the variations of gravity at different latitudes on the bulging and spinning earth. Also, from the differences in the rates at which pendulums swung at different latitudes he was able to derive an approximate shape for the earth.

What Newton had done was to create a whole new quantitative approach to the study of astronomical motions. Because some of his predicted variations had not been observed, new improved instruments were built. These were needed anyway to improve the observations which could now be fitted together under the grand theory. Numerous new theoretical problems also needed attention. For example, what were the predicted and observed interactions of the planets upon their orbital motions? Although the planets are small compared to the sun and are very far apart, their interactions are enough so that masses can be found for Mercury, Venus and Pluto, which do not have satellites. As precise data have accumulated, the Newtonian theory has permitted calculations about the past and future states of the planetary system. For past and future intervals up to some hundreds of millions of years, when the extrapolation becomes fuzzy, the planetary system has been and will be about as it is now.

What amazed Newton's contemporaries and increases our own admiration for him was not only the range and genius of his work in mechanics, not only the originality and elegance of his proofs, but also the detail with which he developed each idea. Having satisfied himself of the correctness of his principle of universal gravitation, he applied it to a wide range of terrestrial and celestial problems, with the result that it became more and more widely accepted. Remember that a theory can never be completely proven; but it becomes increasingly accepted as its usefulness is more widely shown.

See "Universal Gravitation" in Project Physics Reader 2.

The great power of the theory of universal gravitation became even more apparent when others applied it to problems which Newton had not considered. It took almost a century for science to comprehend, verify and round out his work. At the end of a second century (the late 1800's), it was still reasonable for leading scientists and philosophers to claim that most of what had been accomplished in the science of mechanics since Newton's day was but a development or application of his work. Thus, due to the work of Newton himself and of many scientists who followed him, the list of applications of the principle of universal gravitation is a long one.

Q28 What are some of the reasons which caused Newton to comment that "the calculation of the moon's motion is difficult"? (See Fig. 8.10.)

Q29 What were some of the problems and actions that needed further effort as a result of Newton's theory?

8.15 The actual masses of celestial bodies. The full power of the Law of Universal Gravitation could be applied only after the numerical value of the proportionality constant G had been determined. As we noted earlier, although Newton understood the process for determining G experimentally, the actual determination had to await the invention of delicate instruments and special techniques.

The procedure is simple enough: in the laboratory, measure all of the quantities in the equation

$$F_{grav} = G\frac{m_1m_2}{R^2} \qquad (8.4)$$

except G, which can then be computed. The masses of small solid objects can be found easily enough from their weights. Furthermore, measuring the distance between solid objects of definite shape is not a problem. But how is one to measure the small gravitational force between relatively small objects in a laboratory when they are experiencing the large gravitational force of the earth?

This serious technical problem of measurement was eventually solved by the English scientist Henry Cavendish (1731-1810). As a device for measuring gravitational forces he employed a torsion balance (Fig. 8.14), in which the force to be measured twists a wire. This force could be measured separately and the twist of the wire calibrated. Thus, in the Cavendish experiment the torsion balance allowed measurements of the very small gravitational forces exerted on two small masses by two larger ones. This experiment has been progressively improved, and today the accepted value of G is

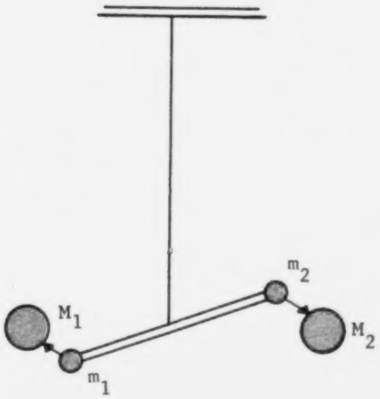


Fig. 8.14 Schematic diagram of the device used by Cavendish for determining the value of the gravitational constant, G. The large lead balls of masses M_1 and M_2 were brought close to the small lead balls of masses m_1 and m_2. The mutual gravitational attraction between M_1 and m_1 and between M_2 and m_2 caused the vertical wire to be twisted by a measurable amount.

$$G = 6.67 \times 10^{-11}\ \mathrm{Nm^2/kg^2}$$

$$= 6.67 \times 10^{-11}\ \mathrm{m^3/kg{\cdot}sec^2}.$$

The gravitational forces between two 1-kg masses such as two quarts of milk one meter apart is less than one ten-billionth of a newton! Evidently gravitation is a weak force which becomes important only when the mass is large.

To derive the gravitational force between two kilogram masses, one meter apart:

$$F = G\left[\frac{m_1m_2}{R^2}\right]$$

$$= 6.67 \times 10^{-11}\left[\frac{\mathrm{Nm}^2}{\mathrm{kg}^2}\right]\left[\frac{1\ \mathrm{kg} \times 1\ \mathrm{kg}}{1\ \mathrm{m}^2}\right]$$

$$= 6.67 \times 10^{-11}\ \mathrm{N}$$

Once the value of G had been determined, the actual masses of the earth, planets and sun could be found. Rearrangement of Eq. (8.9) leads to Eq. (8.12). Since the values of g, R_E and G are now known, the mass of the earth, m_E, can be calculated. Substitute the numbers given in the margin into Eq. (8.12) and find the mass of the earth.

$$G = \left[\frac{R_E{}^2}{m_E}\right]g \qquad (8.9)$$

$$m_E = \left[\frac{R_E{}^2}{G}\right]g \qquad (8.12)$$

$g = 9.80\ \mathrm{m/sec^2}$

$G = 6.67 \times 10^{-11}\ \mathrm{m^3/kg\ sec^2}$

$R_E = 6.37 \times 10^6\ \mathrm{m}.$

Like other large numbers we have encountered, this one is difficult to comprehend. Since at most we can lift but a few kilograms, we have nothing in our experience with which to compare the mass of the earth—or for that matter, the

How to Find the Mass of a Double Star

To demonstrate the power of Newton's laws, let us study a double star. You can even derive its mass from your own observations.

An interesting double star of short period, which can be seen as a double star with a six-inch telescope, is Krüger 60. The find-chart shows its location less than one degree south of the variable star Delta Cephei in the northern sky.

The sequential photographs (Fig. 8.15), spaced in proportion to their dates, show the double star on the right. Another star, which just happens to be in the line of sight, shows on the left. The photographs show the revolution within the double-star system, which has a period of about 45 years. As you can see, the components were farthest apart, about 3.4 seconds of arc, in the mid-1940's. The chart of the relative positions of the two components (Fig. 8.16) shows that they will be closest together at 1.4 seconds of arc around 1971. The circles mark the center of mass of the two-star system. If you measure the direction and distance of one star relative to the other at five-year intervals, you can make a plot on graph paper which shows the motion of one star relative to the other. Would you expect this to be an ellipse? Should Kepler's Law of Areas apply? Does it? Have you assumed that the orbital plane is perpendicular to the line of sight?

The sequential pictures show that the center of mass of Krüger 60 is drifting away from the star at the left. If you were to extend the lines back to earlier dates, you would find that in the 1860's Krüger 60 passed only 4 seconds of arc from that reference star.

The drift of Krüger 60 relative to the reference star shows that the stars do move relative to each other. For most stars, which are at great distances, this motion, called proper motion, is too small to be detected. Krüger 60 is, however, relatively nearby, only about 13 light-years away. (This is the distance light travels in 13 years at 3.0×10^8 meters per second.) The distance to Krüger 60 is then:

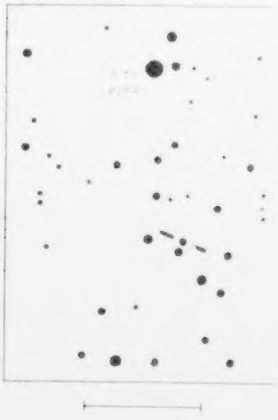
$$13 \text{ light-years} \times 3.0 \times 10^8 \text{ m/sec}$$

$$\times 3.2 \times 10^7 \text{ sec/yr},$$

or $$13 \times 10^{16} \text{ m},$$

or $$8.7 \times 10^5 \text{ A.U.}$$

(One year contains about 3.2×10^7 seconds. One A.U. is 1.5×10^{11} meters.)



A finding chart for Krüger 60, with north upward, east to the left.

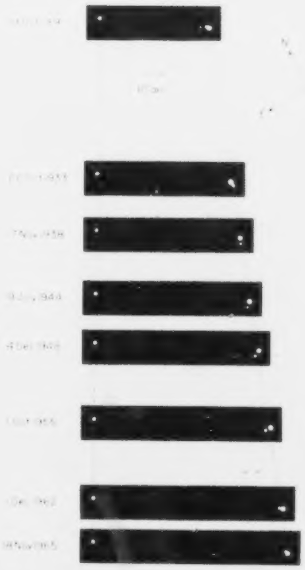


Fig. 8.15 The orbital and linear motions of the visual binary, Krüger 60, are both shown in this chart, made up of photographs taken at Leander McCormick Observatory (1919 and 1933) and at Sproul Observatory (1938 to 1965).

From the sequential photographs and the scale given there we can derive the change in distance of Krüger 60 from the comparison star between 1919 and 1965. From the photographs our measurements give the distances as 55 seconds of arc in 1919 and 99 seconds of arc in 1965. (One second of arc is the angle subtended by a unit length, perpendicular to the line of sight, at a distance of 2.1×10^5 of the units.) Thus the proper motion was 44 seconds in 46 years, very nearly 1.0 second of arc per year. This angle is about $1/2.1 \times 10^5$ of the distance to the star. Then in one year the star moves 13×10^{16} m$/2.1 \times 10^5$, or 6.7×10^{11} m/yr. In one second the component of the star's velocity vector across the sky is 1.9×10^4 m, or its velocity perpendicular to the line of sight is 19 km/sec. Probably the star also has a component of motion along the line of sight, called the radial velocity, but this must be found from another type of observation.

Adapted from paper by James F. Wanner of the Sproul Observatory, Swarthmore College, in Sky and Telescope, January 1967.

The masses of the two stars of Krüger 60 can be found from the photographs shown in Fig. 8.15 and the application of Eq. (8.11). When we developed Eq. (8.11) we assumed that the mass of one body of each pair (sun-planet, or planet-satellite) was negligible. In the equation the mass is actually the sun of the two, so for the double star we must write $(m_1 + m_2)$. Then we have

$$\frac{(m_1 + m_2)\ \text{pair}}{m_{\text{sun}}} = \left[\frac{T_E}{T_{\text{pair}}}\right]^2 \left[\frac{R_{\text{pair}}}{R_E}\right]^3 . \qquad (8.11)$$

The arithmetic is greatly simplified if we take the periods in years and the distances in Astronomical Units (A. U.), which are both unit for the earth. The period of Krüger 60 is about 45 years. The mean distance of the components can be found in seconds of arc from the diagram (Fig. 8.16). The mean separation is

$$\left(\frac{\text{max} + \text{min}}{2}\right) = \frac{3.4\ \text{seconds} + 1.4\ \text{seconds}}{2}$$

$$= \frac{4.8\ \text{seconds}}{2} = 2.4\ \text{seconds}.$$

Earlier we found that the distance from the sun to the pair is near 8.7×10^5 A.U. Then the mean angular separation of 2.4 seconds equals

$$\frac{2.4 \times 8.7 \times 10^5\ \text{A.U.}}{2.1 \times 10^5} = 10\ \text{A.U.},$$

or the stars are separated by about the same distance as Saturn is from the sun. (We might have expected a result of this size because we know that the period of Saturn around the sun is 30 years.)

Now, upon substituting the numbers into Eq. (8.11) we have:

$$\frac{(m_1 + m_2)\ \text{pair}}{m_{\text{sun}}} = \left(\frac{1}{45}\right)^2 \left(\frac{10}{1}\right)^3$$

$$= \frac{1000}{2020} = 0.50,$$

or, the two stars together have about half the mass of the sun.

We can even separate this mass into the two components. In the diagram of motions relative to the center of mass we see that one star has a smaller motion and we conclude that it must be more massive. For the positions of 1970 (or those observed a cycle earlier in 1925) the less massive star is 1.7 times farther than the other from the center of mass.

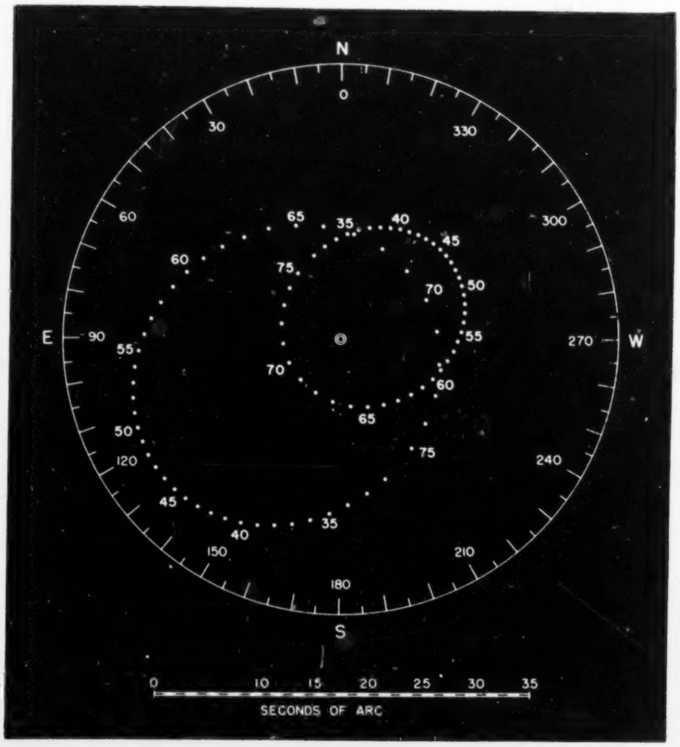


Fig. 8.16 Kruger 60's components trace elliptical orbits, indicated by dots, around their center of mass marked by circles. For the years 1932 to 1975, each dot is plotted on September 1st. The outer circle is calibrated in degrees, so the position angle of the companion may be read directly, through the next decade.

So the masses of the two stars are in the ratio 1.7 : 1. Of the total mass of the pair, the less massive star has

$$\frac{1}{1 + 1.7} \times 0.5 = 0.18 \text{ the mass of the sun,}$$

while the other star has 0.32 the mass of the sun. The more massive star is more than four times brighter than the smaller star. Both stars are red dwarfs, less massive and considerably cooler than the sun.

From many analyses of double stars astronomers have found that the mass of a star is related to its total light output, as shown in Fig. 8.17.

SG 8.15

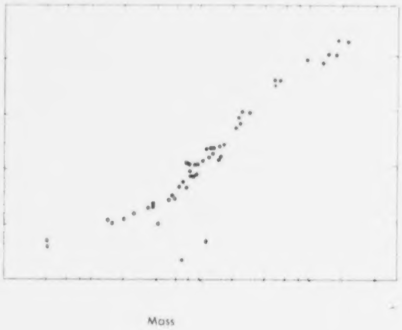


Fig. 8.17 The mass-luminosity relation. The three points at the bottom represent white dwarf stars, which do not conform to the relation.

haps for this reason Cavendish preferred to give his result not as the mass of the earth but as the mass divided by its volume (known quite well from geographical surveys). He concluded that the earth was 5.48 times as dense as water. It is a tribute to his experimental skill that his result is so close to the modern value of 5.52.

This large density for the earth raised questions of very great interest to geologists. The average density of rock found in the crust of the earth is only about 2.7, and even the densest ores found rarely have densities greater than the earth's overall average. Since much mass must be somewhere, and it is not at the surface, apparently the material making up the core of the earth must be of much higher density than the surface material. Some increased density of deep rocks should be expected from the pressure of the upper layers of rock. But squeezing cannot account for most of the difference. Therefore, the center of the earth must be composed of materials more dense than those at the surface and probably is composed mainly of one or more of the denser common elements. Iron and nickel are the most likely candidates, but geologists are also considering other alternatives.

SG 8.13

$$m_S = \frac{4\pi^2 R^3}{GT^2} \qquad (8.10)$$

With the value of G known by experiment we can also find the mass of the sun, or of any celestial object having some type of satellite. Once we know the value of G, we can substitute numbers into Eq. (8.10) and find m_S. In the case of the sun, the earth is the most convenient satellite to use for our computation. The earth's distance from the sun is 1.50×10^{11}m, and its period is one year, or 3.16×10^7 sec. We have already seen that $G = 6.67 \times 10^{-11}\ m^3/kg\cdot sec^2$. Upon substituting these numbers in Eq. (8.10), we find the mass

$$m_S = \frac{4\pi^2(1.50 \times 10^{11}\ m)^3}{(6.67 \times 10^{-11}\ m^3/kg\ sec^2)(3.16 \times 10^7\ sec)^2} = \frac{4\pi^2(1.50 \times 10^{11})^3\ m^3}{(6.67 \times 10^{-11})(3.16 \times 10^7)^2\ \frac{m^3}{kg\ sec^2} \times sec^2}$$

$$m_S = 2.0 \times 10^{30}\ kg$$

of the sun to be 2.0×10^{30} kg. Earlier we found that the mass of the earth was about 6.0×10^{24} kg. Thus, the ratio of the mass of the sun to the mass of the earth is

$$\frac{2 \times 10^{30}\ kg}{6 \times 10^{24}\ kg},$$

which shows that the sun is 3.3×10^5 (about a third of a million) times more massive than the earth.

This same procedure can be used to find the actual mass of any planet having a satellite. For planets not having satellites, their attraction on other planets can be used. But all methods rely at some point on the Law of Universal Gravitation. The masses of the sun, moon and planets relative to the earth are listed in Table 8.2. Notice that the planets taken together add up to not much more than 1/1000th part of the mass of the solar system.

Table 8.2

The mass of the sun, moon, and planets relative to the mass of the earth. The earth's mass is approximately 6.0×10^{24} kg.

Sun	333,000.00	Jupiter	318.
Moon	0.012	Saturn	95.
Mercury	0.056	Uranus	14.6
Venus	0.82	Neptune	17.3
Earth	1.00	Pluto	0.8?
Mars	0.108		

See "Life Story of a Galaxy" in Project Physics Reader 2.

See "Expansion of the Universe" in Project Physics Reader 2.

See "The Stars Within Twenty-two Light Years That Could Have Habitable Planets" in Project Physics Reader 2.

Q30 How was the value of G determined experimentally by Cavendish?

Q31 What geological problems result from the discovery that the mean density of the earth is 5.52?

8.16 Beyond the solar system. We have seen how Newton's laws have been applied to explaining much about the earth and the entire solar system. But now we turn to a new question. Do Newton's laws, which are so useful within the solar system, also apply at greater distances among the stars?

Over the years following publication of the Principia several sets of observations provided an answer to this important question. At the time of the American Revolutionary War, William Herschel, a musician turned amateur astronomer was, with the help of his sister Caroline, making a remarkable series of observations of the sky through his home-made high-quality telescopes. While planning how to measure the parallax due to the earth's motion around the sun, he noted that sometimes one star had another star quite close. He suspected that some of these pairs might actually be double stars held together by their mutual gravitational attractions rather than being just two stars in nearly the same line of sight. Continued observations of the directions and distances from one star to the other of the pair showed that in some cases one star moved during a few years in a small arc of a curved path around the other (see Fig. 8.18). When enough observations had been gathered, astronomers found that these double stars, far removed from the sun and planets, also moved around each other in accordance with

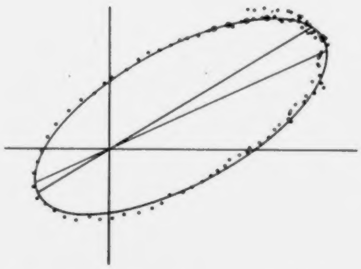


Fig. 8.18 The motion over many years of one component of a binary star system. Each circle indicates the average of observations made over an entire year. Open circles designate the years 1830-1873. Dark circles are used for the years after 1873.

See "Space, The Unconquerable" in Project Physics Reader 2.

p. 102, astronomers have found the masses of these stars range from about 0.1 to 50 times the sun's mass.

Q32 In what ways has the study of double stars led to the conclusion that Newton's Law of Universal Gravitation applies between stars?

8.17 Some influences on Newton's work. The scientific output of Newton and his influence on the science of his time were perhaps unequaled in the history of science. Hence, we should look at Newton as a person and wonder what personal attributes led to his remarkable scientific insight. He was a complicated, quiet bachelor intensely involved with his studies, curiously close to the usual stereotype of a genius, which is so often completely wrong.

Newton was a man of his time and upbringing; some of his work dealt with what we today regard as pseudosciences. He had an early interest in astrology. He seems to have spent much time in his "laboratory," cooking potions that smelled more of alchemy than of chemistry. Yet in all his activities, he seems to have been guided and motivated by a search for simple underlying general principles and never for quick practical gains.

Throughout the discussion above we have been mixing the physics of terrestrial bodies with the motions of celestial bodies, just as Newton did. Since the relationships could be verified, we truly have a synthesis of terrestrial and celestial physics of great power and generality. A brief reconsideration of Eq. (8.4) can remind us of the inclusiveness of the results obtained. If we have

$$F_{grav} = \frac{Gm_1m_2}{R^2}, \qquad (8.4)$$

and if G is actually a universal constant, we are able to derive, and therefore to understand better, many particulars that previously seemed separate. For example, we can conclude:

1. that g should be constant at a particular place on earth;
2. that g might be different at places on earth at different distances from the earth's center, or at different latitudes which have different velocities of daily rotation as the earth turns;
3. that at the earth's surface the weight of a body must be related to its mass;

4. that Kepler's Three Laws of Planetary Motion hold, and are [illegible]

5. that oceanic tides are the result of the net attraction of the sun and moon on the liquid waters.

Throughout Newton's work is his belief that celestial phenomena are explainable by quantitative terrestrial laws. He felt that his laws had a physical meaning and were not just mathematical tricks or conveniences covering unknowable true laws. Just the opposite; the natural physical laws governing the universe were accessible to man, and the simple mathematical forms of the laws were evidence of their reality.

Newton combined the skills and approaches of both the experimental and theoretical scientist. He made ingenious pieces of equipment, such as the first reflecting telescope, and performed skillful experiments, especially in optics. Yet he also applied his great mathematical and logical powers to the creation and analysis of theories to produce explicit, testable statements.

Many of the concepts which Newton used came from his scientific predecessors and contemporaries. For example, Galileo and Descartes had contributed the first step to a proper idea of inertia, which became Newton's First Law of Motion, Kepler's planetary laws were central in Newton's consideration of planetary motions. Huygens, Hooke and others clarified the concepts of force and acceleration, ideas which had been evolving for centuries.

In addition to his own experiments, he selected and used data from a great variety of sources. Tycho Brahe was only one of several astronomers whose observations of the moon's motion he used. When he could not carry out his own measurements, he knew whom to ask.

Lastly, we must recall how fruitfully and exhaustively his own specific contributions were used and expanded throughout his work. For instance, in developing his Theory of Universal Gravitation, his Laws of Motion and his various mathematical inventions were used again and again. Yet Newton was modest about his achievements, and he once said that if he had seen further than others "it was by standing upon the shoulders of Giants."

Q33 What are some of the reasons that Newton is honored today?

8.18 Newton's place in modern science. The success of Newtonian mechanics greatly influenced scientific and philosophical thought in the early eighteenth century. The new mechanistic approach suggested that all observations could be interpreted in terms of mechanical theories. In economics, philosophy, religion and the developing "science of man," the successful approach of Newton and the Newtonians encouraged the rising Age of Reason.

One consequence of the mechanistic attitude, lingering on to the present day, was a widespread belief that with Newton's laws (and later similar ones for electrodynamics) the future of the whole universe and each of its parts could be predicted. One need know only the several positions, velocities and accelerations of all particles at any one instant. As Kepler had suggested, the universe seemed to be a great clockwork. This was a veiled way of saying that everything worth knowing was understandable in terms of physics, and that all of physics had been discovered. As you will see later in this course, in the last hundred years scientists have been obliged to take a less certain position about their knowledge of the world.

Today we honor Newtonian mechanics for less inclusive, but more valid reasons. The content of the Principia historically formed the basis for the development of much of our physics and technology. Also the success of Newton's approach to his problems provided a fruitful method which guided work in the physical sciences for the subsequent two centuries.

We recognize now that Newton's mechanics holds only within a well-defined region of our science. For example, although the forces within each galaxy appear to be Newtonian, one can speculate that non-Newtonian forces operate between galaxies. At the other end of the scale, among atoms and subatomic particles, an entirely non-Newtonian set of concepts had to be developed to account for the observations.

Even within the solar system, there are a few small residual discrepancies between predictions and observations. The most famous is the too great angular motion of the perihelion of Mercury's orbit: Newtonian calculations differ from the observations by some 43 seconds of arc per century.

Such difficulties cannot be traced to a small inaccuracy of the Law of Gravitation, which applies so well in thousands of other cases. Instead, as in the case of the failure of the Copernican system to account accurately for the details of planetary motion, we must reconsider our assumptions. Out of many studies has come the conclusion that Newtonian mechanics cannot be modified to explain certain observations. Newtonian science is joined at one end with relativity theory, which is important for bodies moving at high speeds. At the other end, Newtonian science borders on quantum mechanics, which gives us a physics of atoms, molecules and nuclear particles. But for the vast middle ground, Newtonian mechanics still describes the world of ordinary experience and of classical physics as accurately as it always did.

Q34 What were some of the major consequences of Newton's work on scientists' view of the world?

8.19 *What is a theory*? The making of theories to account for observations is a major purpose of scientific study. Therefore some reflection upon the theories encountered thus far in this course may be useful. In later parts of the course you will recognize many of these aspects of theories, and others too. Perhaps you will come to agree that "there is nothing more practical than a good theory."

A theory is a general statement relating selected aspects of many observations. A theory:

1. *should summarize and not conflict with a body of tested observations*. Examples: Tycho's dissatisfaction with the inaccuracy of the Ptolemaic system, Kepler's unwillingness to explain away the difference of eight minutes of arc between his predictions and Tycho's observations.

2. *should permit predictions of new observations which can be made naturally or arranged in the laboratory*. Examples: Aristarchus' prediction of an annual parallax of the stars, Galileo's predictions of projectile motions and Halley's application of Newton's theory to the motions of comets.

3. *should be consistent with other theories*. Example: Newton's unification of the earlier work of Galileo and of Kepler. However, sometimes new theories are in conflict with others, as for example, the geocentric and heliocentric theories of the planetary system.

Every theory involves assumptions. The most basic are those of Newton's Rules of Reasoning, which express a faith in the stability of things and events we observe. Without such an assumption the world is just a series of happenings which have no common elements. In such a world the gods and goddesses of Greek mythology, or fate, or luck prevent us from finding any regularity and bases for predictions of what will happen in similar circumstances.

Specific assumptions also underlie every theory. In this unit the major assumptions were those of a geocentric or a heliocentric planetary system. The development and gradual acceptance of the heliocentric theory provided a model which Kepler used to interpret the observations of Tycho. Then Newton combined the theories of Galileo and Kepler into one grand theory. Such a vast theory as Universal Gravitation is sometimes called a "grand conceptual scheme."

Making and Judging Theories

How theories are made and judged is perhaps as interesting as a description of what a theory is. No list of operations necessarily applies to the development of all theories. Furthermore, different men proceed in different ways; each has a personal style much like that of an artist. Even so, a partial list of operations in theory making should help you examine other theories which appear in later parts of this course. Do not attempt to memorize this list.

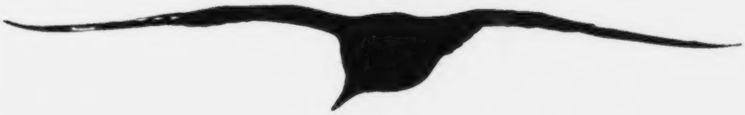
Observations are focused only upon selected aspects of the phenomena. Our interest centers upon the general question: "In what way do the initial conditions result in this reaction?" For example, Galileo asked, "How can I describe the motion of a falling body?"

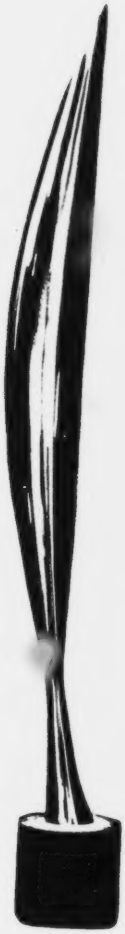
A theory relates many selected observations. A pile of observations is not a theory. Tycho's observations were not a theory, but were the raw material from which one or more theories could be made.

Theories often involve abstract concepts derived from observations. Velocity is difficult to observe directly, but can be found by comparing observations of time and position. Similarly, acceleration is difficult to measure directly, but can be found from other observed quantities. Such abstractions, sometimes called "constructs," are created by scientists as useful ideas which simplify and unite many observations.

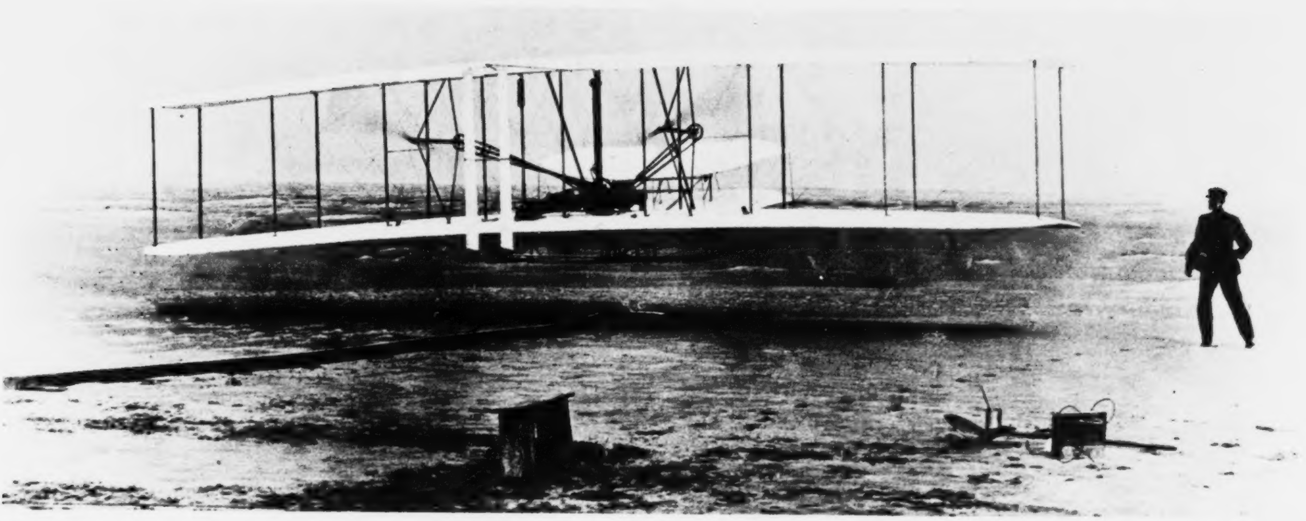
Empirical laws organize many observations and reveal how changes in one quantity vary with changes in another. Examples are Kepler's three laws and Galileo's description of the acceleration of falling bodies.

Theories never fit exactly with observations. The factors in a theory are simplifications or idealizations; conversely, a theory may neglect many known and perhaps other unknown variables. Galileo's theory of projectile motion neglected air resistance.





A bird in flight and two interpretations of flight: Constantin Brancusi's "Bird in Space" (Museum of Modern Art, New York, 1919) and the Wright Brothers' "Kitty Hawk" flight (1903). Analogously, different theories may result from the different intentions of their originators.



Predictions from theories may require new observations. These often require improvements in the precision of apparatus, or the creation of new types of instruments. The annual parallax of the stars predicted by Aristarchus could not be observed until telescopes were invented and developed so that very small angles could be measured reliably.

Theories that have later been discarded may have been initially useful because they encouraged new observations. The idea that comets were some local phenomenon led Tycho to compare distant observations of the directions to a comet.

Theories that permit quantitative predictions are preferred to qualitative theories. Aristotle's theories of motion explained in a rough way how bodies moved, but Galileo's theories were much more precise. The qualitative planetary theory of Tycho and Descarte's vortex theory of motion were interesting, but did not rely upon measurements.

An "unwritten text" lies behind most terms in the seemingly simple statement of theories. For example, "force equals mass times acceleration" is a simple sentence. However, each word carries a specific meaning based on observations and definitions and other abstractions.

Communication between scientists is essential. Scientific societies and their journals, as well as numerous international meetings, allow scientists to know of the work of others. The meetings and journals also provide for public presentations of criticisms and discussions. Aristarchus' heliocentric theory had few supporters for centuries, although later it influenced Copernicus who read of it in the *Almagest*.

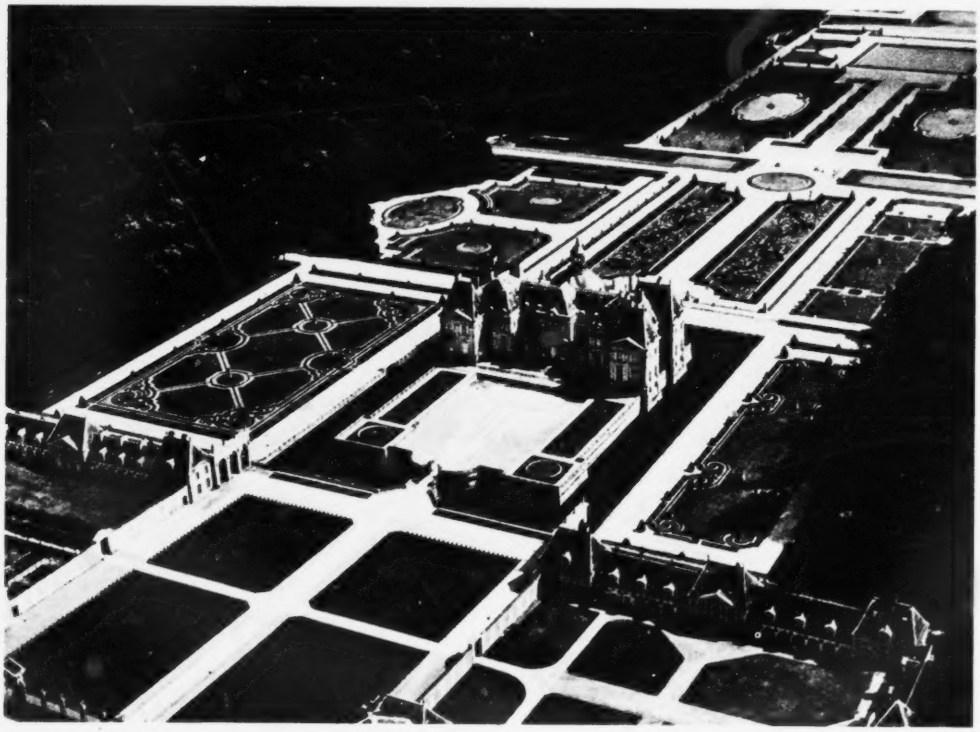
Some theories are so strange that they are accepted very slowly. The heliocentric theory was so different from our geocentric observations that many people were reluctant to accept the theory. Strange theories often involve novel assumptions which only a few men will at first be willing or able to consider seriously. Novelty or strangeness is, of course, no guarantee that a theory is important; many strange theories prove to be quite unuseful.

In the making of a theory, or in its later description to others, models are often used as analogies. Physical models are most easily understood. The chemists' ball-models of atoms are an example. Similarly, a planetarium projects images of heavenly bodies and their motions. Thus models may reproduce some of the phenomena or suggest behaviors predicted by the theory.

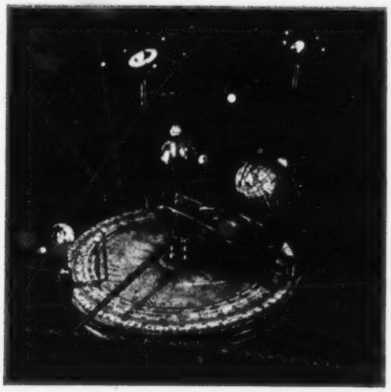
However, models are man-made and are not the real phenomena or the conceptions with which the theory deals in detail. Although models are often quite useful, they can also be misleading. They represent the theory only to the degree that the maker includes some aspects of what he is representing. Also, the maker may add other aspects which do not relate to the original, e.g. chemists' atoms do not have definite sizes like the balls do that serve as visual models.

Other models may be statistical or mathematical. It is useful to consider all scientific theories as models which attempt to describe the suspected interworkings of some quantities abstracted from observations.

The power of theories comes from their generality. Theories are distillates from many observations, empirical laws and definitions. The more precise a theory is, the better it will agree with specific observations. But most important is the usefulness of a theory in describing a wide range of observations and predicting quite new observations. An aesthetic feeling of beauty and niceness, even of elegance is often stimulated by a concise yet broadly inclusive theory.



In areas other than science, conciseness and theory are used to achieve beauty and elegance. Even the gardens around this 17th-century French chateau reflect a preference for order.



See "An Appreciation of The Earth" in Project Physics Reader 2.

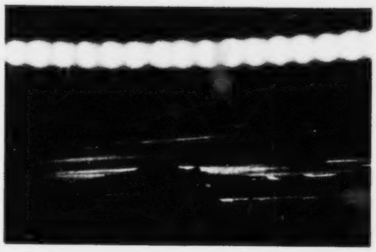
Epilogue In this unit we have reached back to the beginnings of recorded history to follow the attempt of men to explain the regular cyclic motions observed in the heavens. Our purposes were double: to examine with some care the difficulties of changing from an earth-centered view of the heavens to one in which the earth came to be seen as just another planet moving around the sun. Also we wanted to put into perspective Newton's synthesis of earthly and heavenly motions. From time to time we have also suggested the impact of these new world views upon the general culture, at least of the educated people. We stressed that each contributor was a creature of his times, limited in the degree to which he could abandon the teachings on which he was raised, or could create or accept bold new ideas. Gradually through the successive work of many men over several generations, a new way of looking at heavenly motions arose. This in turn opened new possibilities for even further new ideas, and the end is not in sight.

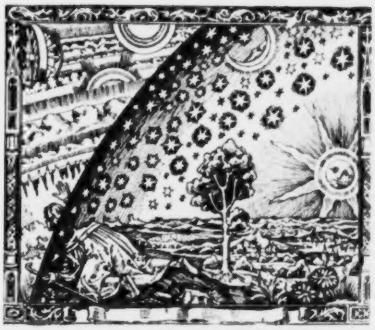
Prominent in our study have been references to scientists in Greece, Egypt, Poland, Denmark, Austria, Italy and England. Each, as Newton said, stood on the shoulders of others. For each major success there are many lesser advances or, indeed, failures. Thus we see science as a cumulative intellectual activity, not restricted by national boundaries or by time; nor is it inevitably and relentlessly successful, but it grows more as a forest grows, with unexpected changes in its different parts.

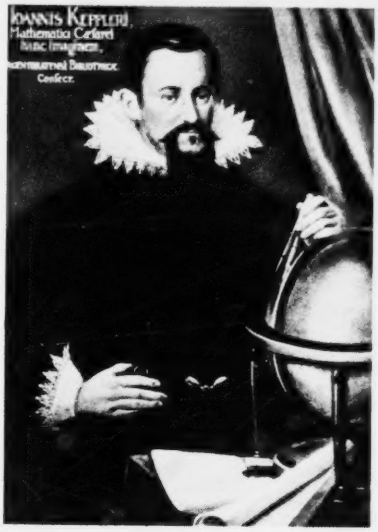
It must also be quite clear that the Newtonian synthesis did not end the effort. In many ways it only opened whole new lines of investigations, both theoretical and observational. In fact, much of our present science and also our technology had its effective beginning with the work of Newton. New models, new mathematical tools and new self-confidence—sometimes misplaced, as in the study of the nature of light—encouraged those who followed to attack the new problems. A never-ending series of questions, answers and more questions was well launched.

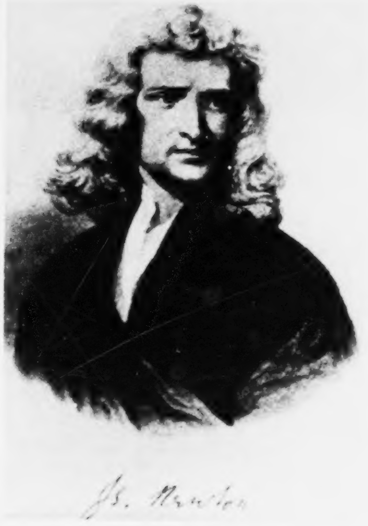
In the perspective of history it is intriguing to speculate why Newton turned to astronomy. Perhaps it was in part because the motions of the planets had been a major and persistent problem for centuries. But at least some of his interest—and reason for success—lay in the fact that the heavenly bodies do not move like those on the earth. In the heavens there is no friction, or air resistance. Thus the possibility that a few simple mathematical relationships between idealized factors could fit observations had its first major application to conditions which were close to the idealized, simplified schemes—and it worked. Then scientists could return to earthly phenomena with renewed confidence that this line of attack could be profitable. We can wonder how mechanics would have developed if we lived on a cloud-bound planet from which the stars and planets were not visible.

Among the many problems remaining after Newton's work was the study of objects interacting not by gravitational forces but by friction and collisions. Experiments were soon to raise questions about what aspects of moving bodies were really important. This led, as the next unit shows, to the identification of momentum and of kinetic energy, and then to a much broader view of the nature and importance of energy. Eventually from this line of study emerged other statements as grand as Newton's Universal Gravitation: the conservation laws on which so much of modern physics—and technology—is based, especially the part having to do with many interacting bodies making up a system. That account will be introduced in Unit 3.









Development of Equations in Chapter 8.

$F_{grav} \propto \frac{1}{R^2}$, initially an assumption

(8.1) $F_{grav} \propto m_P m_S$, a conclusion from the definition of force

(8.2) $F_{grav} \propto \frac{m_P m_S}{R^2}$, combines the two relations above

(8.3) $F_{grav} = G\frac{m_P m_S}{R^2}$, relation (8.2) restated as an equation by inclusion of constant G

(8.4) $F_{grav} = G\frac{m_1 m_2}{R^2}$, Law of Universal Gravitation for any two masses: m_1, m_2.

(8.5) $a_c = \frac{v^2}{R}$, centripetal acceleration, from Unit 1

but

$v = 2\pi R/T$,

and

$v^2 = 4\pi^2 R^2/T^2$,

so

(8.6) $a_c = 4\pi^2 R/T^2$.

However, because

$F = ma$,

(8.7) $F_c = ma_c$

$= m4\pi^2 R/T^2$, the centripetal force

SG 8.14

SG 8.17

Equate (8.3) and (8.7),

$F_{grav} = F_c$,

(8.8) $T_P{}^2 = \left[\frac{4\pi^2}{Gm_S}\right]R^3$

Rearrange (8.8)

(8.10) $m_S = \left(\frac{4\pi^2}{G}\right)\left(\frac{R^3}{T_P{}^2}\right)$

Compare two examples of (8.10) for planet and satellite

(8.11) $\frac{m_{sun}}{m_P} = \left[\frac{T_{sat}^2}{T_P{}^2}\right]\left[\frac{R_P{}^3}{R_{sat}^3}\right]$

$F_{grav} = \text{Weight}$

$G\frac{m_E m}{R^2} = mg$, which becomes

(8.9) $G = \left[\frac{R_E{}^2}{m_E}\right]g$,

or

(8.12) $m_E = \left[\frac{R_E{}^2}{G}\right]g$

8.1 If the velocity of a planet is greater at A than at B, where is the sun located? In what direction does the vector difference of the velocities at A and B point?

8.2 a) Compare the force on the earth due to the sun's gravitational attraction with the force on the sun due to the earth's gravitational attraction.

b) If the mass of the sun is $3.3 \times 10^5 \times$ the mass of the earth, compare the acceleration of the sun toward the earth with the acceleration of the earth toward the sun.

8.3 Draw a rough graph of the weight of a 1-kilogram mass in terms of its distance from the center of the earth. Take the radius of the earth as R and go out to 5R. Points at 2R, 3R, 4R, and 5R would be sufficient. Will the weight be zero at 100R? At 1000R?

8.4 Two bodies, A and B, are observed to be moving in circular orbits. The orbital radius and period of body A are both twice those of body B. Could both bodies be moving around the sun? Explain your conclusion.

8.5 If the radius of planet A's orbit is twice that of planet B's, what is the ratio of (a) their periods, (b) their orbital speeds, (c) their accelerations toward the sun?

8.6 Two satellites revolve around different planets at the same distance, R. One satellite has a period three times that of the other.

a) Which planet has the larger mass?
b) What is the ratio of the masses of the two planets?

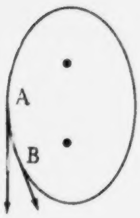
8.7 At what altitude above the earth's surface would the acceleration due to gravity be

a) 3/4 g?
b) 1/2 g?
c) 1/4 g?

8.8 a) What is the acceleration due to gravity on the moon's surface? [Hint: Equate the weight of any mass m_1 on the moon with the force exerted on it according to the Universal Law of Gravitation.] The moon's radius is 1738 km, and its mass is 7.15×10^{22} kg.

b) How much would a 72-kg astronaut weigh on the moon's surface? What would be his mass there?

8.9 Mass of Jupiter: 1.90×10^{27} kg. Average distance from Jupiter to sun: 7.78×10^8 km.



a) At what point on an imaginary line connecting the sun's center with the center of Jupiter would a spacecraft have no net force from these two bodies?
b) How does the distance between this point and the center of Jupiter compare with the mean orbital radius of the orbit of Jupiter's outermost satellite (orbital radius = 2.37×10^7 km)?
c) From your results in parts (a) and (b) what peculiarities, if any, would you expect to observe in the satellite's motion?
d) Would you expect to find a satellite even farther from Jupiter?

8.10 If two planets in another solar system have the same average density, but the radius of one of them is twice that of the other:

a) which one would have the greater surface gravitational pull?
b) what is the ratio between the surface gravitational strengths of the two planets?

8.11 Mars has two satellites, Phobos and Deimos—Fear and Panic. The inner one, Phobos, revolves at 5,800 miles from the center of Mars with a period of 7 hrs 39 min.

a) Since the rotation period of Mars is 24 hrs 37 min, what can you conclude about the apparent motion of Phobos as seen from Mars?
b) What is the mass of Mars?

8.12 The shortest earth-Mars distance is about 56×10^6 km; the shortest Mars-Jupiter distance is about 490×10^6 km. The masses of these planets are as follows:

Earth	5.96×10^{24} kg
Mars	6.58×10^{23} kg
Jupiter	1.91×10^{27} kg.

a) When (under what conditions) do these "shortest" distances occur?
b) What is the gravitational force between the earth and Mars when they are closest together? Between Jupiter and Mars?
c) Do you expect the motion of Mars to be influenced more by the attraction of the earth or of Jupiter?

8.13 Callisto, the second largest satellite of Jupiter, is observed to have a period of revolution of 1.442×10^6 sec. Its mean orbital radius is 1.87×10^6 km. Using only these data and the value of the Universal Gravitational Constant, calculate the mass of Jupiter.

8.14 Making use of the expression $\frac{GM}{R^2} = \frac{v^2}{R}$, which follows directly from the Law of Gravitation, show that the time taken per revolution by a satellite at a distance R from the center of the earth is given by:

$$T = 2\pi\left(\frac{R^3}{GM}\right)^{\frac{1}{2}}.$$

Hint: make use of $T = \frac{2\pi R}{v}$.

8.15 Two balls, each having a mass m, are separated by a distance r. Find the point which lies along the line joining their centers where the gravitational attractions cancel one another.

a) What will happen to this point if both the balls are made twice their size?
b) What will happen to this point if one of the balls is made twice as heavy as the other?

8.16

Mass of the moon:	7.18×10^{22} kg
Mass of the sun:	1.99×10^{30} kg
Mass of the earth:	5.96×10^{24} kg
Average distance from sun to earth:	1.495×10^{8} km
Average distance from moon to earth:	3.84×10^{5} km.

a) Derive the gravitational pull of the moon, and of the sun on the earth.
b) Does the sun or the moon have the greatest effect on the earth's tides? Can you explain this quantitatively?

8.17 At the earth's surface, a 1-kg mass standard weighs 9.81 newtons. The Midas 3 satellite launched July 27, 1964 orbits in a nearly circular orbit 3,430 km above the earth's surface with a period of 161.5 minutes. What is the centripetal acceleration of the satellite? What is the condition for a circular orbit? (Remember, the radius of the satellite's orbit is the height above the ground plus the earth's radius.)

Answers to End of Section Questions

Your self-checking answers need not be as elaborate as these.

Chapter 5

Q1 We conclude that the ancient peoples watched the skies from: cave paintings of star patterns, the orientation of the pyramids in Egypt, Stonehenge and similar structures in England, Scotland and France; the alignment of observing windows in buildings in Mexico and Peru. Written records were made in Babylon, Greece, Egypt, China since some centuries B.C.

Q2 Calendars were needed for establishing the proper times for agricultural activities and for religious rites.

Q3 During one year the sun shows three motions: daily rising and setting, seasonal drift eastward among the stars, and seasonal north-south variation.

Q4 The difference between the Gregorian calendar and the positions of the sun will add up to one day during an interval of 3,333 years.

Q5 During a month, which begins with new moon, as the moon passes the sun, the moon continually moves eastward but not at an exactly even rate. The moon also moves north and south so that it is always near the ecliptic.

Q6 Usually when the moon passes the sun, or the direction opposite to the sun, the moon is north or south of the ecliptic. Thus the moon's shadow misses the earth at new moon. Similarly, the moon moves above or below the earth's shadow at full moon. However, twice a year the moon is near the ecliptic at new and full moon. These are the times when eclipses of the sun and moon can occur.

Q7 Mercury and Venus are always observed near the sun. They will be low in the west after sunset, or low in the east before sunrise.

Q8 When in opposition a planet is opposite the sun. Therefore the planet would rise at sunset and be on the north-south line at midnight.

Q9 Mercury and Venus show retrograde, that is, westward motion among the stars, after they have been farthest east of the sun and visible in the evening sky. At this time they are brightest, and nearest the earth. They are then moving between the earth and sun.

Q10 Mars, Jupiter and Saturn show retrograde motion when they are near opposition.

Q11 The retrograde motion of Mars has the largest angular displacement, but the shortest period. Saturn has the longest period of the planets visible to the naked eye, but has the smallest angular displacement. The retrograde motion of Jupiter is intermediate in displacement and duration.

Q12 Plato assumed that the motions of the planets could be described by some combination of uniform motions along circles. He also assumed that the earth was at the center of the largest circle for each planet.

Q13 Our knowledge of Greek science, as well as that of every other ancient civilization, is incomplete because many of their written records have been destroyed by fire, weathering and decay. Yet each year new records are being unearthed and deciphered.

Q14 The Greeks around the time of Plato assumed that a theory should be based on self-evident propositions. Quantitative observations were rarely used as a basis for judging the usefulness of a theory.

Q15 A geocentric system is an earth-centered system. It is also an observer-centered system, because as observers we are on the earth.

Q16 The first solution to Plato's problem was made by Eudoxus. He described the planetary motions by a system of transparent crystalline spheres which turned at various rates around various axes.

Q17 Aristarchus assumed that the earth rotated daily—which accounted for all the daily motions observed in the sky. He also assumed that the earth revolved around the sun—which accounted for the many annual changes observed in the sky.

Q18 If the earth were moving around the sun, it would have a shorter period than would Mars, Jupiter and Saturn. When the earth moved between one of these planets and the sun (with the planet being observed in opposition), the earth would move faster than the planet. We would see the planet moving westward in the sky as retrograde motion.

Q19 The distance between the earth and sun was known to be some millions of miles. If the earth revolved around the sun during a year, the direction to the stars should show an annual shift—the annual parallax. This was not observed until 1836 A.D.

Q20 Aristarchus was considered to be impious because he suggested that the earth, the abode of human life, might not be at the center of the universe.

Q21 Ptolemy assumed: 1) that the heaven is spherical and rotates once each day around the earth, 2) that the earth is spherical, 3) that the earth is at the center of the heavens, 4) that the size of the earth is negligible in comparison to the distance to the stars, 5) that the earth has no motions, and 6) that uniform motion along circles is the proper behavior for celestial objects.

Q22 If the earth rotated, Ptolemy argued that birds would be left behind and that great winds would continually blow from the east.

Q23 The radii of the epicycles of Mars, Jupiter and Saturn must always be parallel to the line between the earth and sun. This also meant that each of these epicycles had a period of exactly one year.

Q24 The Ptolemaic system was purely a mathematical model and probably no one believed it was a physical description.

Q25 Ptolemy displaced the earth from the exact center of the universe with his equants and eccentrics.

Q26 The Ptolemaic system survived because it predicted the positions of sun, moon and planets; because it agreed with the philosophical and theological doctrines and because it made sense, and it was not challenged by a better, simpler model.

Chapter 6

Q1 Copernicus rejected the use of equants because a planet moving on an equant did not move at a uniform angular velocity around either the center of the equant or around the earth. This was essentially an aesthetic judgment.

Q2 Apparently Copernicus meant that the sun was not exactly at the center of motion for any planet (he used eccentrics). Yet in general the sun was in the center of all the various planetary motions.

Q3

	Assumed by	
	Ptolemy	Copernicus
a) The earth is spherical	Yes	Yes
b) The earth is only a point compared to the distances to the stars	Yes	Yes
c) The heavens rotate daily around the earth	Yes	No
d) The earth has one or more motions	No	Yes
e) Heavenly motions are circular	Yes	Yes
f) The observed retrograde motion of the planets results from the earth's motion around the sun	No	Yes

Q4 Copernicus derived distances to the planetary orbits in terms of the earth's distance from the sun. He also derived periods for the planetary motions around the sun. So far as we know, Aristarchus did not develop his heliocentric theory to the point that he reached similar results.

Q5 Copernicus argued that his heliocentric system was inherently simpler than the geocentric system of Ptolemy. Also, his results agreed at least as well as Ptolemy's with observations. Furthermore, Copernicus argued that his simple system reflected the mind of the Divine Architect.

Q6 The predictions made by both Ptolemy and Copernicus differ from observations by as much as 2°, four diameters of the moon.

Q7 Copernicus argued that the distance to the stars must be very great because they showed no annual shift (parallax). But the distance required to make the parallax unobservable was enormously greater than people wanted to accept. That the predicted shift was not observed was weak evidence. Negative evidence is not very convincing even when there seem to be only two possible alternative conclusions.

Q8 Simple is a difficult word to interpret. The actual computational scheme required by the Copernican system was not simple, even though the general idea of a moving earth and a fixed sun seemed simple.

Q9 As the quotation from Francis Bacon indicates, the evidence did not permit a clear choice between the two possible explanations. To many people the argument seemed to be a tempest in a tea cup.

Q10 The major difference between the Ptolemaic and the Copernican systems was the assumption about the mobility of the earth: daily rotation and annual revolution about the sun. This was a difference in the frame of reference.

Q11 Because the astronomical interpretations dealt with the structure of the whole universe, they overlapped with conclusions drawn from religious discussions. The synthesis by Thomas Aquinas of Aristotelean science and Christian theology increased the difficulty of discussing one separately from the other.

Q12 The Copernican system conflicted with the accepted frame of reference in which the earth was central and stationary. The distances which Copernicus derived for the separation of the planets from the sun implied that vast volumes of space might be empty. This conflicted with the old assumption that "nature abhors a vacuum."

Q13 Some conflicts between scientific theories and philosophical assumptions are:

a) the earth appears to be very old although strict interpretation of Biblical statements lead to ages of only a few thousand years, less than those attributed to ancient cities;
b) the geological assumption of uniformitarianism—slow changes throughout long times—conflicted with the assumption of abrupt changes in the earth's history from major floods, earthquakes, etc.;
c) the idea of slow genetic evolution in biology clashed with the idea of a unique and recent creation of mankind;
d) the astronomical conclusion that the earth was only one of several planets clashed with the assumption that the earth was created uniquely as the abode of human life.

Q14 Copernicus brought to general attention the possibility of a new explanation of the astronomical observations. This shift in assumptions permitted others—Kepler, Galileo and later Newton—to apply and expand the initial propositions of Copernicus.

Q15 The Copernican system proposed that the earth was just one of many planets and was in no way uniquely created as the abode of life. Therefore, there might be life on other planets.

Currently astronomers conclude that there might be some form of life on Mars or Venus. The other planets in our solar system seem to be too hot or too cold, or to have other conditions opposed to life-as-we-know-it. There is now general agreement that many other stars probably have planets and that life might well exist on some of them. A clear distinction must be made between some form of life and what we call intelligent life. Since the only means by which we could learn of any intelligent life on planets around other stars would be through radio signals, intelligent life means that the living organisms would transmit strong radio signals. On earth this has been a possibility for less than half a century. Thus we have, in these terms, been "intelligent life" for only a few years.

Q16 Tycho's observations of the new star and then of the comet of 1577 directed his attention to astronomical studies.

Q17 Tycho's conclusions about the comet of 1577 were important because the comet was shown to be an astronomical body—far beyond the moon. Also the comet moved erratically, unlike the planets, and seemed to go through the crystalline spheres of the Aristotelean explanation.

Q18 Tycho's observatory was like a modern research institute because he devised new instruments and had craftsmen able to make them, he had a long-term observing program, he included visitors from other places, and he worked up and published his results.

Q19 The Bayeaux tapestry shows the people cowering below tne image of Halley's comet. Other pictures may show similar scenes. Many allusions to the dire effects of comets appear in Shakespeare and other writers.

Q20 For several months during 1909-1910 Halley's comet moved westward—in retrograde motion. Because it stayed near the ecliptic, we might suspect that its orbital plane makes only a small angle with the earth's orbital plane (the ecliptic plane). Whether the comet is moving in the direction of the planets or in the opposite direction is not clear. In Chapter 8 the motion of this comet is mentioned. Also there is an Optional Activity which leads to an explanation of the motion observed in 1909-1910.

Q21 Tycho made instruments larger and stronger. Also he introduced the use of finer scales so that fractions of degrees could be determined more accurately.

Q22 Tycho corrected his observations for the effects of atmospheric refraction. He established the amount of the corrections by long series of observations of objects at various angular distances above the horizon.

Q23 The Tychonic system had some features of both the Ptolemaic and the Copernican systems. Tycho held the earth stationary, but had the planets revolving around the sun, which in turn revolved around the earth.

Q24 Whether the Copernican system in its entirety could be interpreted as a "real" system of planetary paths in space is doubtful. If the minor cycles required to account for small observed variations were neglected, the major motions might be considered to represent "real" orbits. Copernicus did not discuss this aspect of his system.

Chapter 7

Q1 Tycho became interested in Kepler through Kepler's book, in which he tried to explain the spacing of the planetary orbits by the use of geometrical solids.

Q2 After some 70 attempts with circles, eccentrics and equants, Kepler still had a difference of 8' between his best prediction and Tycho's observed positions for Mars. Kepler finally decided that no combination of circular motions would yield a solution. (He might have been wrong.)

Q3 Kepler used the observations made by Tycho Brahe. These were the most accurate astronomical observations made up to that time.

Q4 First Kepler had to refine the orbit of the earth. Then he could use the earth's position to triangulate positions of Mars in its orbit.

Q5 Kepler discovered that the planets moved in planes which passed through the sun. This eliminated the necessity to consider the north-south motions of each planet separately from its eastward motion.

Q6 Kepler's Law of Areas: the line from the sun to the moving planet sweeps over areas that are proportional to the time intervals.

Q7 Kepler noted the direction of Mars when it was in opposition. He knew that after 687 days Mars was at the same point in its orbit, but the earth was at a different place. By reversing the directions observed from the earth to Mars and to the sun, he could establish positions on the earth's orbit.

Q8 The component of a planet's velocity perpendicular to the line from the sun to the planet changes inversely with the distance of the planet from the sun.

Q9 When a circle is not viewed from directly above its center, it has an elliptical shape.

Q10 Of the naked-eye planets which Tycho could observe, Mars has an orbit with the largest eccentricity. If the orbit of Mars was considerably less eccentric, Kepler would not have found that its orbit was an ellipse. He could not have found that any of the other orbits were ellipses.

Q11 Kepler knew that after 687 days Mars had returned to the same point in its orbit. Observations from the earth at intervals of 687 days provided sightlines which crossed at the position of Mars on its orbit.

Q12 Kepler's Law of Periods: the squares of the periods of the planets are proportional to the cubes of their mean distances from the sun. If the distance or period of a planet is known, the other value can be computed.

Q13 Kepler compared the "celestial machine" to a gigantic clockwork.

Q14 Kepler's reference to a "clockwork" is significant because it suggests a "world machine." This idea was developed as a result of Newton's studies.

Q15 According to the Ptolemaic theory Venus was always between the earth and the sun. Therefore, it should always show a crescent shape. However, Galileo observed that it showed all phases, like the moon. Therefore, to show full-phase Venus had to pass behind the sun. Such positions for Venus were consistent with either the heliocentric or the Tychonic system.

Q16 With the telescope Galileo discovered the phases of Venus—which were contrary to the Ptolemaic system. He also discovered the system of satellites around Jupiter—a miniature Copernican system, but NOT around the earth or the sun. Thus the earth was not the only possible center of motion, as the Ptolemaeans had contended. Galileo's telescopic observations of the moon, the sun, Saturn and the stars were interesting, but not critically related to the heliocentric model.

Q17 Galileo's observation of the satellites of Jupiter showed that there could be motions around centers other than the earth. This contradicted basic assumptions in the physics of Aristotle and the astronomy of Ptolemy. Galileo was encouraged to continue and sharpened his attacks on those earlier theories.

Q18 Kepler and Galileo emphasized the importance of observations as the raw material which must be explained by theories.

Q19 Galileo was said to be impious. Also he was sharp in his criticisms of others and often used ridicule. Officially he was tried for breaking his agreement not to support the Copernican system as really correct.

Chapter 8

Q1 Newton's isolation on the farm during 1665-66 provided time for him to contemplate a variety of scientific questions.

Q2 Scientific societies provided a forum where ideas and experimental procedures could be discussed. Publications from the societies allowed scientists at a distance to learn of the various studies reported. Today the scientific societies perform all these services. They also sponsor international meetings and international cooperative activities.

Q3 Newton said that he was "contemplative" when he saw the apple fall. He wondered about what he saw and began to seek for explanations.

Q4 You write your own answer to this one.

Q5 The *Principia* was originally written in Latin. Even the translations are difficult to read because Newton adopted a very mathematical style with complicated geometrical proofs.

Q6 a) Orbits are ellipses + Newtonian inertia —> there is a net force acting.

b) Step (a) + Kepler's Law of Areas around sun + Newton's Law of Areas around center of force —> sun must be at the center of force.

c) For elliptical orbits (or any conic section) around sun at one focus, $F \propto 1/R^2$.

d) $F \propto 1/R^2$ —> ? Law of Periods. Yes, only this force law will satisfy Kepler's Law of Periods.

Q7 Kepler's Law of Areas is satisfied by any central force when the areas are measured from the center of force.

Q8 Only Kepler's Law of Periods provides information about the behavior of planets which have different mean distances from the sun. Their motions allow us to study how the gravitational force from the sun changes with distance.

Q9 The derivation assumed that: a) Galileo's law of acceleration applied to planetary motions as well as to falling stones; b) that the strength of the gravitational force varies as $1/R^2$; c) that the planets move in circular orbits. We also assumed that the strength of the gravitational force does not depend upon direction, time (date) or the relative velocities of the bodies.

Q10 Newton's Law of Universal Gravitation predicts accurately an enormous number of observed changes.

Q11 Scientists could not imagine how one object could influence another through great distances in empty space. The question of "action at a distance" comes up again in Unit 4.

Q12 Newton did not publicly attempt to explain gravitation because he could not discover any causes from the observations. Possible "causes" of gravitation are still being discussed.

Q13 We do not observe the motion of the earth toward a falling stone for two reasons: 1) we are on the earth and would rise with it—that is, we see only the change in the stone's position; also 2) the mass of the stone is so minute compared to the mass of the earth that the motion of the earth would be undetectable.

Q14 The Constant of Universal Gravitation, G, is a number by which we can equate observed accelerations (and forces) with the masses and distances supposedly responsible for those accelerations. So far as we can tell, the value of G does not change with position in the universe or with the passage of time.

Q15 A universal law, like any other scientific law, can be tested only in a limited number of cases. The more varied the cases are, the more confident we can be about the usefulness of the law.

Q16 A mass point is a concept by which we consider all the mass of a spherical homogeneous body to be concentrated at the center of the body.

Q17 Even though Newton did not know the value of G, he could form ratios of quantities with the result that the value of G would cancel between the numerator and the denominator. Thus Newton could obtain relative masses of the planets and the sun.

Q18 Newton concluded that the moon was accelerated toward the earth just as an apple was. He compared the observed and computed accelerations of the moon and found that "they agreed tolerably well."

Q19 Newton, like other scientists, could think up many possible explanations for what is observed. Numerical results indicate whether or not the ideas fit reasonably well with observations.

Q20 Newton compared the centripetal force on the moon, based on observations, with the value he extrapolated from the acceleration of falling apples and stones.

Q21 Newton concluded that the value of G could be found from Eq. (8.9):

$$G = \frac{R_E^{\,2}}{m_E}\,g,$$

in which the size and mass of the earth, R_E and m_E, were accepted as constant. From numerous studies he concluded that, for a given place, g was constant. Then G must be constant.

Q22 No. The value of g, the gravitational acceleration at a particular place, is expected to differ between places. As Newton showed, because of the earth's rotation g depends upon latitude on the earth. It also depends upon elevation, or distance from the earth's center. Local variations in the materials of the rocks, the presence of nearby canyons or peaks and other variations also affect the value of g at a particular point. But if g is constant at a point, then G is also constant.

Q23 We consider the solid earth to be a mass point responding all over to the moon's attraction at the earth's center. Although the solid earth bends a bit under the differences in the moon's attractions on the near and far sides, this change is much less than the effects of the moon's pull on the fluid waters, which move readily under the moon's changing attraction.

Q24 On the side of the earth away from the moon, the attraction of the moon is less than it is upon the center of the solid earth. As a result, the earth is pulled away from the fluid water on the side away from the moon and there is a net force away from the moon.

Q25 Newton had explained the motions of the planets, which moved in nearly circular orbits. Halley then showed that the same force laws would explain the motions of comets, which previously had been considered as unexplainable. Some of the comets, moving in very elongated ellipses, even had periods not much greater than Saturn (30 years).

Q26 By forming ratios of two expressions like Eq. (8.10), Newton could compare the masses of the sun and each planet which had a satellite of known period. These planets were, in Newton's time, the earth, Jupiter and Saturn.

Q27 We can compute the mass of Uranus compared to the mass of the sun by using Eq. (8.11),

$$\frac{m_{sun}}{m_p} = \left[\frac{T_{sat}}{T_p}\right]^2 \left[\frac{R_p}{R_{sat}}\right]^3$$

$T_{sat} = 1^d 10^h = \frac{1.416\ d}{365\ d/y} = 3.88 \times 10^{-3}\ y$

$T_p \;\; = 84.0\ y$

$R_{sat} = 81{,}000\ mi = 8.71 \times 10^{-4}\ A.U.$

$R_p = 19.19\ A.U.$

$$\frac{m_{sun}}{m_p} = \left[\frac{3.88 \times 10^{-3}\ y}{84.0\ y}\right]^2 \left[\frac{19.19\ A.U.}{8.71 \times 10^{-4}\ A.U.}\right]^3$$

$$= \left[4.62 \times 10^{-5}\right]^2 \left[2.20 \times 10^4\right]^3$$

$$= 21.3 \times 10^{-10} \times 10.65 \times 10^{12}$$

$$= 22{,}700.$$

Since the mass of the sun is 332,000 times the mass of the earth,

$$\frac{m_{\text{Uranus}}}{m_{\text{earth}}} = \frac{332{,}000}{22{,}700} = 14.6.$$

Q28 The moon's orbit around the earth is somewhat eccentric (e = 0.055). During its cycles of the earth, the moon's distance from the sun also changes. As a result the accelerations on the moon and therefore the orbit of the moon change continuously. This is an example of the "three-body" problem for which no general solution has yet been developed.

Q29 Following Newton's work careful observations were needed to determine the magnitude of many small variations which his theory predicted. Also the general problem of the interactions between the planets had to be worked out.

Q30 Cavendish used a torsion balance to measure the gravitational attraction between metal spheres.

Q31 Geologists and astronomers were obliged to explain how the mean density of the earth could be 5.52, which is much greater than the density of surface rocks. Apparently the earth has a central core of high density.

Q32 The motions of double stars can be explained quantitatively by Newton's Theory of Gravitation. Apparently the gravitational force between such stars is identical to that between the sun and planets.

Q33 Newton is honored today for his development of the Theory of Universal Gravitation, for his development of the mathematics called the infinitesimal calculus, for his work in optics, for his creation of the first reflecting telescope, and for his services to the Royal Society and to his national government.

Q34 After Newton the idea of a great World Machine was widely accepted for several hundred years. More emphasis was put upon precise observations to determine the rate at which various predicted changes occurred. How to account for gravitation and other "actions at a distance" became an important problem. Scientists felt confident that their observations were revealing a "real world" which could be explained in terms of mechanical systems.

Chapter 5

5.1 Discussion
5.2 Discussion
5.3 Discussion
5.4 Discussion

Chapter 6

6.1 (a) (b) Discussion
    (c) 87.5 days
    (d) Discussion
    (e) 224 days

Chapter 7

7.1 89.5 years
7.2 (a) 18 A.U.
    (b) 1.8 A.U.
    (c) 0.053
7.3 4%
7.4 249 years
7.5 0.594
7.6 Discussion

Chapter 8

8.1 Upper focus
8.2 3×10^{-6}
8.3 An activity
8.4 No
8.5 (a) 2.8
    (b) 0.71
    (c) 0.25
8.6 (a) Shorter period
    (b) 9:1
8.7 (a) 0.15
    (b) 0.41
    (c) 1.00
8.8 (a) 1.58 m/sec^2
    (b) 114 N
8.9 (a) 2.33×10^{7} km
        7.55×10^{8} km
    (b) About the same
    (c) Discussion
    (d) No
8.10 (a) Larger one
    (b) 2:1
8.11 (a) Discussion
    (b) 6.7×10^{23} kg
8.12 (a) Discussion
    (b) 8.3×10^{16} N
        3.5×10^{17} N
    (c) Discussion
8.13 1.86×10^{27} kg
8.14 Derivation
8.15 (a) The same point
    (b) 1.41
8.16 (a) 1.93×10^{20} N
        3.52×10^{22} N
    (b) Discussion
8.17 (a) 4.2 m/sec^2

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Prologue

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Chapter 5

Fig. 5.2 Emil Schulthess, Black Star Publishing Company, Inc.

Fig. 5.3 John Stofan.

Fig. 5.4 John Bufkin, Macon, Missouri, Feb. 1964.

Pp. 9, 10 Mount Wilson and Palomar Observatories.

Fig. 5.9 DeGolyer Collection, University of Oklahoma Libraries.

Chapter 6

P. 26 Deutsches Museum, Munich.

Fig. 6.1 Woodcut by Sabinus Kauffmann, 1617. Bartynowski Collection, Cracow.

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Chapter 7

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P. 49 Kepler, Johannes, Mysterium cosmographicum, Linz, 1596.

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Fig. 7.13 Instituto e Museo di Storia della Scienza, Florence, Italy.

Fig. 7.14 DeGolyer Collection, University of Oklahoma Libraries.

Fig. 7.17 Lowell Observatory Photograph.

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Chapter 8

P. 74 Yerkes Observatory.

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PLANETS AND THEIR SATELLITES

The Planets

Name		Symbol	Mean Distance from Sun		Period of Revolution		Eccentricity of Orbit	Inclination to Ecliptic
			Astron. Units	Million Miles	Sidereal	Synodic		
					days	days		
Inner	Mercury	☿	0.3871	35.96	87.969	115.88	0.206	7° 0′
	Venus	♀	0.7233	67.20	224.701	583.92	0.007	3 24
	Earth	⊕	1.0000	92.90	365.256	.....	0.017	0 0
	Mars	♂	1.5237	141.6	686.980	779.94	0.093	1 51
					years			
	Ceres	①	2.7673	257.1	4.604	466.60	0.077	10 37
Outer	Jupiter	♃	5.2028	483.3	11.862	398.88	0.048	1 18
	Saturn	♄	9.5388	886.2	29.458	378.09	0.056	2 29
	Uranus	♅	19.1820	1783	84.015	369.66	0.047	0 46
	Neptune	♆	30.0577	2794	164.788	367.49	0.009	1 46
	Pluto	♇	39.5177	3670	247.697	366.74	0.249	17 9

Name	Mean Diameter in Miles	Mass ⊕ = 1	Density Water = 1	Period of Rotation	Inclination of Equator to Orbit	Oblateness	Stellar Magnitude at Greatest Brilliancy
Sun ☉	864,000	331,950	1.41	$24^d.65$	7° 10′	0	−26.8
Moon ☾	2,160	0.012	3.33	27 .32	6 41	0	−12.6
Mercury	2,900	0.05	6.1	88	7?	0	−1.9
Venus	7,600	0.81	5.06	30?	23?	0	−4.4
Earth	7,913	1.00	5.52	$23^h\ 56^m$	23 27	1/296	......
Mars	4,200	0.11	4.12	24 37	24	1/192	−2.8
Jupiter	86,800	318.4	1.35	9 50	3 7	1/15	−2.5
Saturn	71,500	95.3	0.71	10 02	26 45	1/9.5	−0.4
Uranus	29,400	14.5	1.56	10 45	98	1/14	+5.7
Neptune	28,000	17.2	2.29	15 48?	29	1/40	+7.6

The Satellites

Name	Discovery		Mean Distance in Miles	Period of Revolution	Diameter in Miles	Stellar Magnitude at Mean Opposition
Moon			238,857	$27^d\ 7^h\ 43^m$	2160	−12
Satellites of Mars						
Phobos	Hall,	1877	5,800	0 7 39	10?	+12
Deimos	Hall,	1877	14,600	1 6 18	5?	13
Satellites of Jupiter						
Fifth	Barnard,	1892	113,000	0 11 53	150?	13
I Io	Galileo,	1610	262,000	1 18 28	2000	5
II Europa	Galileo,	1610	417,000	3 13 14	1800	6
III Ganymede	Galileo,	1610	666,000	7 3 43	3100	5
IV Callisto	Galileo,	1610	1,170,000	16 16 32	2800	6
Sixth	Perrine,	1904	7,120,000	250 14	100?	14
Seventh	Perrine,	1905	7,290,000	259 14	35?	17
Tenth	Nicholson,	1938	7,300,000	260 12	15?	19
Twelfth	Nicholson,	1951	13,000,000	625	14?	19
Eleventh	Nicholson,	1938	14,000,000	700	19?	18
Eighth	Melotte,	1908	14,600,000	739	35?	17
Ninth	Nicholson,	1914	14,700,000	758	17?	19
Satellites of Saturn						
Mimas	Herschel,	1789	115,000	0 22 37	300?	12
Enceladus	Herschel,	1789	148,000	1 8 53	350	12
Tethys	Cassini,	1684	183,000	1 21 18	500	11
Dione	Cassini,	1684	234,000	2 17 41	500	11
Rhea	Cassini,	1672	327,000	4 12 25	1000	10
Titan	Huygens,	1655	759,000	15 22 41	2850	8
Hyperion	Bond,	1848	920,000	21 6 38	300?	13
Iapetus	Cassini,	1671	2,210,000	79 7 56	800	11
Phoebe	Pickering,	1898	8,034,000	550	200?	14
Satellites of Uranus						
Miranda	Kuiper,	1948	81,000	1 9 56	.....	17
Ariel	Lassell,	1851	119,000	2 12 29	600?	15
Umbriel	Lassell,	1851	166,000	4 3 28	400?	15
Titania	Herschel,	1787	272,000	8 16 56	1000?	14
Oberon	Herschel,	1787	364,000	13 11 7	900?	14
Satellites of Neptune						
Triton	Lassell,	1846	220,000	5 21 3	2350	13
Nereid	Kuiper,	1949	3,440,000	359 10	200?	19

Glossary

The following is a list of words that appear in the text, but which may not be familiar to the average reader.

action at a distance. The process in which one body exerts a force on another without any direct or indirect physical contact between the two.

angular altitude. The angle measured in degrees that a star, planet, the sun or the moon appears above the horizontal.

angular motion. The orbital motion of a planet, satellite or star, measured in degrees per unit time.

angular size. The angle subtended by an object. For instance, the sun, the moon and your thumbnail at arm's length have about the same angular size.

aphelion. The point of an orbit that is furthest from the sun.

arc length. The distance along an arc or orbit.

astrology. The study of the supposed influences of the stars and planets on human affairs (e.g., horoscopes).

Astronomical Unit (A.U.). The average distance between the earth and the sun.

atmospheric refraction. The bending of light rays that occurs when light enters the atmosphere at an acute angle.

cause. An event or relationship that is always followed by another particular event or relationship; the absence of the first implies the absence of the second, all other conditions being equal.

celestial. Of the sky or the heavens.

celestial equator. The great circle formed by the intersection of the plane of the earth's equator with the celestial sphere.

celestial sphere. The imaginary spherical shell containing the stars and having the earth as a center.

center of motion (or force). The point toward which an orbiting body is accelerated by a central force; the point toward which a central force is everywhere directed.

central force. A force that is always directed towards a particular point, called the *center of force*.

centripetal acceleration. Acceleration of an orbiting body toward the center of force.

comet. A luminous celestial body of irregular shape having an elongated orbit about the sun.

conic section. Any figure that is the total intersection of a plane and a cone. The four types are the circle, ellipse, parabola and hyperbola.

Constant of Universal Gravitation (G). The constant of proportionality between the gravitational force between two masses, and the product of the masses divided by the square of the distance of separation. (See also *gravitation*.)

constellation. One of eighty-eight regions of the sky; the star pattern in such a region.

cyclic variation. A change or process of change that occurs in cycles, i.e., that repeats itself after a fixed period of time, such as the phases of the moon.

deferent. The circle along which the center of an epicycle moves.

density. Mass per unit volume.

double star. A pair of stars that are relatively close together and in orbit about each other.

eccentric. A circular orbit with the earth displaced from the center, but on which the orbiting body moves with constant velocity.

eclipse. The shadowing of one celestial body by another: the moon by the earth, the sun by the moon.

ecliptic. The yearly path of the sun against the background of stars. It forms a great circle on the celestial sphere.

ellipse. A closed plane curve generated by a point moving in such a way that the sums of its distances from two fixed points is a constant. The two points are called the *foci* of the ellipse. If the two points are the same, then the figure is a circle, and the foci are its center.

empirical. Based on observation or sense experience.

empirical law. A statement that concisely summarizes a set of experimental data or observations. Example: Kepler's *Law of Areas*.

epicycle. Part of the geometrical construction formerly used to describe the motions of celestial bodies; it is the small circle whose center moves along the deferent.

equant. A circle on which the orbiting body moves with constant angular velocity about a point different from the center of the circle, and such that the center of the circle is midway between that point and the earth.

equatorial bulge. The bulging of a planet or star at its equator, due to the flattening effect of the body's rotation.

equinox. A time when the sun's path crosses the plane of the earth's equator. The vernal equinox occurs on approximately March 21, the autumnal equinox on approximately September 22.

essences. The four basic elements thought by the Greeks to compose all materials found on the earth. They are: earth, water, air and fire.

experimental philosophy. The term formerly used for *experimental science*; the systematic study of the external world based on observation and experiment.

focus. (See ellipse.)

frame of reference. The coordinate system, or pattern of objects, used to describe a position or motion.

geocentric. Having the earth as a center.

geologist. One who studies the material composition of the earth and its changes, both past and present.

gravitation. The attractive force that every pair of masses exerts on each other. Its magnitude is proportional to the product of the two masses and inversely proportional to the square of the distance between them.

gravity. The gravitational force exerted by the earth on terrestrial bodies; gravitation in general.

Halley's comet. A very bright comet, whose orbital period (about 75 years) was first discovered by the astronomer Edmund Halley.

heliocentric. Having the sun (helios) as a center.

hypothesis. An idea that is tentatively proposed as a basis for a theory; a statement that is not yet accepted as true, because either it has not been proven for enough cases, or it fails to explain *all* the observed phenomena.

interaction. Action on each other, such as by a mutual attractive force, or a collision.

inverse-square force. A central force whose magnitude at any point is inversely proportional to the square of the distance of that point from the force center.

law. A prediction that under specified circumstances, a certain event will always occur, or a certain relationship will be true.

light year. The distance traveled by a ray of light in one year (about 5.88×10^{12} mi.)

mass-point. The center of gravitational force of an object with mass. (See *center of force*.)

model. A combination of physical or mathematical devices that represents and suggests an explanation for the behavior of some actual physical system. Synonym: mechanism.

natural motion. The Aristotelian concept that objects have a motion that is a property of the object itself and therefore the natural way in which the object moves. For example, the Aristotelians thought that the sun's natural motion was to revolve daily about the earth, that the natural motion of fire was to rise, of earth, to fall.

natural place. The concept in Aristotelian science that every object has a particular location where it belongs, and that if an object is removed from this proper location, it will tend to move back there. Thus gravity was explained by saying that objects fall to the surface of the earth because this is their natural place.

Newtonian synthesis. Newton's system of mechanics, which proposed that the same laws govern both terrestrial and celestial motions.

North Celestial Pole. An imaginary point on the celestial sphere about which the stars appear (to an observer in the northern hemisphere) to rotate. This apparent rotation is due to the rotation of the earth.

occult quality. A physical property or cause that is hidden from view, mysterious or undiscoverable, but whose existence is assumed in order to explain certain observed effects.

opposition. The moment when a body is opposite the sun in the sky, i.e., when the earth is between that body and the sun, and lies on the line joining them.

orb. A sphere; usually refers to the imaginary crystalline spheres formerly thought to move the planets. Synonym: spherical shell.

orbit. The path of a celestial body that is revolving about some other body.

pantheism. The belief that God is the same as nature or the physical universe.

parallactic shift. The quantity of apparent change in position of an object due to a change in position of the observer. (See parallax.)

parallax. The apparent displacement of an object due to a change in position of the observer.

perihelion. The point of an orbit that is nearest the sun.

period. The time taken by a celestial object to go once completely around its orbit.

perspective geometry. The study of how three-dimensional objects in space appear when projected onto a plane.

phases of the moon. The variations of the observed shape of the sunlit portion of the moon during one complete revolution of the moon about the earth.

phenomenon. An observed, or observable event.

physical cause. An action of one body on another by direct or indirect physical contact.

Principle of Parsimony. Newton's first "Rule of Reasoning" for framing hypotheses, which asserts that nature is essentially simple in its processes, and therefore that the hypothesis that explains the facts in the simplest manner is the "truest" hypothesis.

Principles of Unity. Newton's second and third "Rules of Reasoning," which assert that similar effects have similar causes, and that if every experiment confirms a certain hypothesis, then it is reasonable to assume that the hypothesis is true universally.

quadrant. A device for measuring the angular separation between stars, as viewed from the earth.

qualitative. Pertaining to descriptive qualities of an object, texture, color, etc.

quantitative. Involving numbers, or properties and relationships that can be measured or defined numerically. Weight is a quantitative property of an object, but texture is not.

quintessence. The fifth basic element or "essence" in the Greek theory of materials, supposed to be the material of which all celestial objects are composed.

regular geometrical solid. A three-dimensional closed figure whose surface is made up of identical regular plane polygons.

retrograde motion. The apparent backward (i.e., westward) motion of a planet against the background of stars that occurs periodically.

satellite. A small body that revolves around a planet (such as the moon around the earth).

Scholastics. Persons who studied and taught a very formal system of knowledge in the Middle Ages, which based truth on authority (the teachings of Aristotle and the Church fathers), rather than on observation.

solar year. The interval of time between two successive passages of the sun through the vernal equinox.

statistics. The study dealing with the classification and interpretation of numerical data.

sun-spots. Relatively dark spots that appear periodically on the surface of the sun. They are presumably caused by unusually turbulent gases that erupt from the sun's interior, and cool rapidly as they reach the surface, which causes them to appear darker than the surrounding solar matter.

systematic error. Error introduced into a measurement (e.g., by inaccuracies in the construction or use of the measuring equipment), whose effect is to make the measured values consistently higher or lower than the actual value.

terrestrial. Pertaining to the earth, or earthly.

theory. A system of ideas that relates, or suggests a causal connection between certain phenomena in the external world. (See cause.)

torsion balance. An instrument which measures very small forces by determining the amount of twisting they cause in a slender wire.

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